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*The Carus Mathematical Monographs*

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# A HISTORY OF MATHEMATICS IN AMERICA BEFORE 1900

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# INTRODUCTION

When we consider the increase of interest in the study of mathematics in this country since the year 1900, and the recent achievements in this field, we may be tempted to feel that the subject had no history of any moment before the twentieth century. Even a brief examination of the question, however, shows not only that the work done before the year 1875 is worthy of attention, but that the succeeding quarter of a century saw laid the foundations upon which the scholars of today have so successfully built.

It was because of my belief in the importance of the subject that I accepted the invitation extended by the Carus Monograph Committee to undertake the work. I made the condition, however, that I should be allowed to join with me Professor Ginsburg, whose ability to search out mathematical facts in libraries is unusual and who, as I had known from years of experience, would attack the problem in complete sympathy with my own methods. As a result of our desire to consult original sources whenever possible, we have spent a much longer time upon ascertaining the facts, in sorting out what seemed the most important, in arranging the material, and in preparing the text, than would seem necessary to one not accustomed to such an undertaking. I wish, therefore, to say that to Professor Ginsburg I am indebted for a painstaking examination of source material in several of our largest libraries, and that to

the several librarians we are both indebted for their generous assistance in the search.

In a general way the plan has been to consider the term "America" as including that territory north of the Caribbean Sea and the Rio Grande River. The changes in the governments of large portions of this territory are of no importance from the standpoint of the history of mathematics, and the fact that more than half of it is a part of the British Commonwealth has no bearing upon the scientific achievements of the two political divisions. Within this "America" it has been our aim to consider (1) the early colonies—their racial inheritances, their early needs in the field of mathematics, and their achievements, meager though they were; (2) the changes in foreign influence from time to time, particularly with respect to the influx of mathematical ideas from the British Islands, France, Germany, and Italy; (3) the beginning of high-grade university work and the rapid development of this work towards the close of the nineteenth century; (4) the development of means for encouraging mathematical progress through the forming of societies and through the publication of the results of investigation in appropriate periodicals; and (5) the names and achievements of a number of those who have, through opening new fields or cultivating old ones, contributed in some worthy manner to the advance of the science.

Naturally, it was possible to make use of only a small part of the material accumulated, the space allowed us being strictly limited. For this reason no attempt could be made to give anything like a complete list of scholars whose primary interest in any period was mathematics,

to make any exhaustive presentation of the changing courses of study, to include complete lists or any analysis of papers published here or abroad in scientific journals, or to devote much space to biography or bibliography. Fortunately there are extended lists of arithmetics prepared by Professor L. C. Karpinski, and of algebras by Professor Lao G. Simons, besides others in the same or similar lines, and hence it was unnecessary to attempt what had already been so effectively accomplished. Neither would it have been desirable to cover in any detail the Spanish achievements already studied by the late Professor Cajori. On the other hand, it was manifestly desirable to pay considerable attention to notable publications which have had an influence upon mathematics in the United States and Canada, and occasionally to speak of works of less importance which show the lower levels of achievement.

As to articles which have appeared in mathematical journals, it has been impossible to present at length any discussion of the contents of even those of importance. All that authors of a work of this size can be expected to do, in this respect, is to mention such monographs and articles, written before 1900, as seem to have laid the foundation for the noteworthy achievements in mathematics in this or other countries since the opening of the twentieth century.

As to the close union of mathematics, physics, and astronomy in the early period, it is difficult to draw the lines which separate the three, and indeed it is undesirable to attempt to do so. While it has been the aim to confine the work rather closely to the somewhat undefinable domain of pure mathematics, it has in a num-

ber of cases been necessary to devote considerable space to the early development of the related sciences.

A difficulty arises in connection with the work of those scholars now living who contributed in some marked degree, at the close of the nineteenth century, to the remarkable progress of mathematics after the year 1900. Necessarily the most important of their contributions have been mentioned, but biographical notes concerning the authors and their later achievements have been left for future historians.

The authors wish to express their thanks to the Fogg Museum, Harvard University, for permission to use photographs from oil paintings of John Winthrop and Benjamin Peirce, and to Columbia University for similar permission to use one of Robert Adrain. The other portraits are from photographs or engravings.

DAVID EUGENE SMITH

January 1934

## CONTENTS

*Abbreviations*

The following abbreviations are generally used in the footnotes and bibliographies. A few others appear occasionally but will be understood readily without further explanation.

*Am. J. M.*, American Journal of Mathematics
*Am. Math. Mo.*, American Mathematical Monthly
*Annals*, Annals of Mathematics
*Bibl. Math.*, Bibliotheca Mathematica
*Bull. Am. M. S.*, Bulletin of the American Mathematical Society
*Bull. N.Y. M. S.*, Bulletin of the New York Mathematical Society
*Encyk. M. W.*, Encyklopädie der Mathematischen Wissenschaften
*Jahrbuch*, Jahrbuch über die Fortschritte der Mathematik
*Messenger*, Messenger of Mathematics
*Proc. Am. Acad.*, Proceedings of the Amer. Acad. of Arts and Sciences
*Quarterly Journ.*, Quarterly Journal of Mathematics, pure and applied
*Trans. Am. M. S.*, Transactions of the American Mathematical Society

The Roman numerals refer to volumes; the modern numerals to pages.

# CHAPTER I

## THE SIXTEENTH AND SEVENTEENTH CENTURIES

### 1. NEEDS OF THE EARLY SETTLERS

The early settlers in America were chiefly British, Dutch, Swedish, French, and Spanish. The Dutch had as the nucleus of their possessions Manhattan Island, soon surrendered to the British. The Swedes, with the region arround Philadelphia as their New Sweden, also gave way to the English settlers. The French finally took control of Lower Canada and held that territory politically until the British acquired it by the treaty of 1763, and they still hold much of it racially. The region known as Louisiana, much larger than the present state, was held by France (1682–1762, 1800–1803) or Spain (by treaty, 1762; actually, 1769–1800) until acquired by the United States in 1803. Florida had been explored by the Spanish in the 16th century, had received a band of Huguenots in 1562, and after various vicissitudes was made a part of the United States in 1821. These events all have a bearing upon the problem to be considered.

In the first place, what were the immediate needs of these early settlers for a subject like mathematics? In answering this question it is desirable to consider their occupations. In general they tended to become small farmers or artisans or to go into trade. A relatively small number were concerned with the sea and with trade with the mother countries. Many were influenced to migrate to the New World by religious persecution, fan-

cied or real, and these settlers developed various christian sects which at the present time seem, to say the least, rather unnecessary to faith in the essentials of Christianity. These sects seem to have been often characterized by bigotry rather than by a desire to love their neighbors as themselves. In each community there were doubtless found men with some medical knowledge, and women to act as midwives.

As to mathematics, the farmer and artisan needed merely the ability to add and subtract numbers, rarely to multiply, and almost never to divide. Their simple purchases and sales were largely matters of barter, and the arithmetic which was needed was easily passed on from father to son, along with the ability to read parts of the Bible. The girls did not, in general, learn anything about calculation. Boys going into trade needed to know but little about computation except how to write a column of figures (often in Roman numerals before the seventeenth century and even later), to add the numbers, and to make simple subtractions, all of which were learned by them in their apprenticeship stage. Most boys, therefore, needed a slight knowledge of simple computation, of the measures in common use, and of halves, fourths, and eighths, together with English or other European monies, most of the latter being based upon the scale used by the British. The land owner needed some knowledge of surveying, but at first this did not concern the average citizen.

This leaves the sea-faring men and the clergy, and it was to them that mathematics appealed for two distinct reasons. The navigators of the 13th century, particularly in the Mediterranean Sea, needed to know how

to find their way from port to port by means of the compass and the portolano maps which gave the courses and which illustrated the *Portolani* or sailing directions. In crossing the Atlantic the early voyagers could find their approximate latitude by the aid of the quadrant or the astrolabe, but they had no satisfactory way of finding their longitude except by dead reckoning. It was not until the second half of the 18th century that the Harrison Chronometer was perfected sufficiently to allow for any close approximation to the longitude of a ship. Hence the amount of mathematics needed to navigate a ship before the 18th century was small. The British Nautical Almanac did not appear until 1767. In the 16th and 17th centuries, therefore, the science of navigation, which in the 18th century developed into an important branch of mathematics, was such as to demand but little beyond simple angle measurement.

It may be thought that the clergy and the healers of one kind or another would have little need for any mathematics beyond simple measurements and calculation. It was in their fields, however, that the science may be said to have had its beginning in the New World. The clergy were divided into two very distinct branches, —the Catholics (which at the time of the first settlers may for our purposes be said to have included the Church of England) and the Protestants. The former paid great attention to the fixing of Easter and other movable feasts, and the latter generally opposed the observation of such festivals as being of papal origin. Each, however, needed calendars for business purposes. The result was that the entire clergy, as being the most highly educated of the people, were called upon to de-

cide matters concerning the calendar, and the Catholics naturally assumed the preparation of the one used in their churches, with the fixing of Easter, Christmas, and the various saints' days. It is here that we find the first trace of mathematics worth considering in the New World, and the mathematics which received the greatest amount of attention before the 19th century. Out of it grew the demand for observatories and for courses in astronomy in the early colleges. The fact that the Catholic churches were so much concerned with such questions as the fixing of Easter led the educators of the early missionaries, chiefly in the French and Spanish colonies, to include a degree of training in astronomy which rendered them able to furnish the information needed in calendar making. In particular, the Jesuit order, founded in 1539, put forth a determined effort to make its priests proficient in this science, an effort which later resulted in producing a larger number of worthy astronomers and a considerable number of notable scholars in this field, and in establishing well-equipped observatories in various parts of the world.

As to the medical men, they were the scientists of the early period and in general were acquainted with, if not believers in, astrology. The 16th century was a time in which this subject was still held in high esteem all over Europe, with the natural result that it found its way into America in this and the period immediately following. With it there came a demand, limited though it was, for enough astronomical knowledge to meet the needs of those who could cast horoscopes and who assumed to prognosticate events. Out of this demand there came the calendars of the 17th century with the vain fore-

casts which are still found in almanacs intended for the lower levels of intelligence. We have, then, one source from which the slender stream of mathematics in the 16th century flowed—the spring of astronomy.

One feature of more decided mathematical interest, however, should be mentioned—the surveying of large tracts of land. One of the early evidences of this interest is seen in the sending out of Thomas Harriot (1560–1621) by Sir Walter Raleigh in 1585 for the purpose of surveying and mapping the southern portion of the territory then known as Virginia but which is now North Carolina. Harriot had taken his B.A. degree at Oxford at the age of nineteen and was only twenty-five when he accompanied Sir Richard Granville (1585) to the New World. A year after his return (1587) to England he published a report upon the colony.[1] His work on algebra (posthumous, London, 1631) set a much higher standard for the subject in England, but it was written more than twenty years after his visit to America. He was also an astronomer of unusual ability, but no trace of his mathematical or astronomical attainments, aside from his survey, is found in the American literature of the century of his death.

It is evident that Harriot could not have been the only man who visited the colonies for the purpose of mapping the newly discovered region. Such a large territory required the work of many who knew at least the rudiments of surveying, and we shall presently see that men of much prominence in public affairs were often skilled in this art.

[1] On his map see P. L. Phillips, *Virginia Cartography*. Washington, 1896.

## 2. CAUSES OF THE LOW DEGREE OF GENERAL INTELLECTUAL EFFORT

In the sixteenth century, however, the general intellectual equipment of the surveyors seems to have been low, and the mathematical attainments of the people were meager. In the year 1814 De Witt Clinton (1769–1828), one of the leading statesmen of his time and a scientist of repute, gave an address in which he set forth with great frankness and clarity the causes of the failure of America to make any progress in the arts and sciences during the entire colonial period. These causes were so well considered by him as to make a brief summary of them appropriate before proceeding to a statement of the successes and failures in the development of mathematics in the New World. As Clinton conceived them they were as follows:

1. Our pioneers came to this country primarily to acquire wealth, expecting to return to their native lands when this was accomplished. Their loyalty was to their country of origin, not to that of their adoption.

2. They came at a time when the intellectual world was involved in philosophies of words, concerned only with logical subtleties which led to the general neglect of science.

3. The colonial governors showed, in general, no interest in the welfare of the country.

4. That desire for fame which encourages genius was stifled through the lack of an enlightened public.

5. Racial and religious prejudices rendered impossible any combination of effort to advance science. This is seen in the attitude of the Dutch in New Amsterdam, the French in Canada and Illinois, the Germans in Pennsylvania, and the British in New England.

6. The fact that the parent countries shipped criminals and other undesirable persons to this country prejudiced Europe against the character and possible attainments of our people.

The combined effects of these causes led to a general lack of school facilities and to a debased condition of the learned professions. No printing press was established in the Dutch provinces until 1693, and no newspaper was published until 1728.

### 3. EARLY CONDITIONS IN THE SEVENTEENTH CENTURY

That the seventeenth century in America could not be expected to make any worthy contribution, even to the most elementary phase of mathematics, is apparent from a brief statement of the situation in the colonies. The Dutch traders hardly began their work until 1611, and not until 1625 did their settlements on Manhattan Island (the present New York) and at Fort Orange (the present Albany) assume any importance. The first school on Manhattan Island was established about 1633–1637, probably with Adam Roelantson[1] as master, and even after the British took final possesion (1673) they established only two grammar (Latin) schools.

During the Dutch supremacy, closing in 1664 (except for the year 1673–1674), only Latin and the vernacular received any attention in the government schools, although schoolmasters sent out by the West Indies Company were required also to teach arithmetic. The salary of a schoolmaster at this time was augmented by 50 stivers (about $1.00) quarterly from each pupil taught to read, write, and "cipher." Under the Dutch control an academy seems to have been planned in 1650 and a Latin school was conducted by Curtius and Luyck, pupils coming from as far away as Virginia. Of one of

[1] This may have been 1637 when his license was issued, although 1633 is often given.

the private tutors, one Jacques Cortelyou, it is said that he was versed "in the mathematics of Descartes," that he "spoke Latin and good French," was a surveyor, and a "good cartesian and not a good Christian, regulating himself in all externals by reason and justice solely." Aside from the simplest primary arithmetic it seems probable that the only mathematics taught during the Dutch colonial period referred to surveying and map making, these being the most essential items in a new country. About ten cents a week was paid for each pupil who learned reading, while writing and "ciphering" cost about 28 cents a week.[2] The learned class was composed largely of clergymen, not of teachers, the attention of the early settlers being given to religious rather than to academic instruction.

The facilities for studying arithmetic in and about New Amsterdam were pitifully few. The pupils rarely saw a printed textbook on the subject, their only aids being an occasional hornbook with the numerals, and later the "battledore" booklets.

### 4. NEW ENGLAND

In New England, however, the outlook was more promising. Although the territory did not receive its name until 1614 and its "Pilgrims" until 1620, or begin its representative government until 1634, as early as 1636 it saw the first American college, Harvard, founded.

[2] Imogen Clark, *Old Days and Old Ways*, New York, 1928; W. H. Kilpatrick, *The Dutch Schools in New Netherlands and Colonial New York*, Washington, 1912. In our discussion we shall speak of elementary calculations as arithmetic and hence as a branch of mathematics, a relatively late usage without early historical sanction, but one which is now generally permitted.

At that time the entire population of New England was only about 4000, and Boston was a village of only twenty-five or thirty houses, but it is a significant fact that as many as one hundred men in the colony had been educated at Oxford or Cambridge, eighty being graduates, and that twenty graduates from Scotch universities were to be found among the settlers.[1]

Since Harvard, as indeed all the early American colleges, was designed primarily for the training of the clergy, it followed the English plan of confining its work chiefly to Latin and Greek. The courses in scientific subjects in the seventeenth century, even for the master's degree, were trivial. Indeed, the only mathematical thesis offered before 1700 was—"Is the quadrature of a circle possible?" in which the candidate took the affirmative. This was in 1693.[2]

The fact that Oxford and Cambridge accepted Harvard degrees in the seventeenth century as equivalent to their own has little significance when we consider that this was done only in the period 1648–1660 when the Puritans were in power in England and wished to favor the Puritan colony in the New World. Educationally, there was no reason for the concession, even as regards Latin and Greek alone.

Meanwhile New England was finding a competitor in the South. In 1660 the Virginia Assembly enacted that land should be furnished for a college and free school, but the matter rested until, in 1688, the necessary money was secured by private subscription. In

[1] J. M. Barker, *Colleges in America*, Cleveland, 1894.
[2] E. J. Young, *Subjects for Master's Degree in Harvard College, 1655–1794*, Cambridge, Mass., 1880.

1692 the first royal educational charter in America was given and a year later William and Mary College was established at Williamsburg, supported by public funds.[3] Here, as at Harvard, no attention was given to mathematics, unless introductory courses in arithmetic may be included in this term.

Thus the encouragement to mathematics given by the colonial governments in the seventeenth century amounted to nothing, and the achievements in the colleges were of no moment. There remains, however, that somewhat imponderable factor—the individual. For a long period mathematics made its great progress through individuals rather than schools. Tartaglia, Cardan, Vieta, Fermat, Descartes, L'Hospital, D'Alembert, and many other giants in the field had little or no relation to any university. It is only in recent times that universities have sought to give to men of this stamp any place on their faculties. So it is not in the schools that the advance in mathematics in the sixteenth, seventeenth, and eighteenth centuries in America is to be found, but rather in the quasi-astronomical work of the astrologers and the makers of almanacs, the two classes often merging in one.

## 5. EARLY ASTRONOMY

The English settlers were familiar with the almanacs published in the seventeenth century by the Stationers' Company, which claimed the monopoly, and at the close of the century with the famous *Vox Stellarum* of Francis Moore (1700). But long before this publication appeared there were almanac makers, astrologers, and

[3] Barker, *loc. cit.*, p. 41.

astronomers of some merit at work in the colonies. For example, Samuel Danforth (1626–1674) of Harvard College, who published "An Almanack" in Boston in 1647, was an astronomer of local distinction in New England. He published later "An astronomical description of the late comet. . . ." (1664), privately printed at Cambridge, Mass., in 1665,[1] and Urian Oakes (1631–1681), the fourth president of Harvard, published at Cambridge a series of astronomical calculations.[2] Even Danforth's, however, was not the first of the American almanacs, for one had already been published by "William Peirce, mariner," at Cambridge in 1639, apparently the second item published by Stephen Daye who had brought his press with him in 1638.

A number of these almanacs appeared in this country in the seventeenth century, but the most scholarly work was done by Thomas Brattle whose book was printed in Cambridge, Mass., in 1678. The British astronomer, Francis Baily (1731–1810) tells us that it was Brattle to whom Newton refers in the *Principia* as having made unusually good observations on the comet of 1682. His first work appeared under the title *Almanack of Coelesstial Motions of the Sun and Planets with Their Principal Aspects for the Year 1678*. He was a member of the Harvard faculty, and his description of his work on the solar eclipses of 1692 and 1703, whereby the difference of longitude between Cambridge, Mass., and London was determined, was published in the *Philosophical Transactions*, London, XXIV (1706), pp. 1630–1638.

[1] The original is in the library of the Massachusetts Historical Society. See *Scripta Mathematica*, I, 275.

[2] J. L. Chamberlain, *Universities and Their Sons*, vol. II, p. 225.

In these observations he was assisted by his pupil, Henry Newman, who also published (1690) an almanac.[3]

It should be understood, however, that most of these almanacs were mere "prognostications," a term frequently used in connection with them. Unlike the church calendars they were not concerned primarily with Easter, nor with saints' days, nor with such scientific knowledge as the prediction of eclipses. They generally pretended to forecast the weather, doing so in a vague manner designed to avoid the criticism of failure. Such "farmers' almanacs" were common until the close of the nineteenth century.

The church calendar was usually announced by the catholic priests, and directions for finding Easter are still given in some editions of the Episcopal Prayer Book. Although based only upon medieval astronomy they were still too scientific for popular use.

## 6. THE ASTROLOGERS

As to the astrologers a brief note will suffice. They often went out with the early settlers as was the case in all migrations of peoples in historic times. For example, the immigrants from Sweden preceded William Penn by a whole generation, erecting a fort on Minquas River, near Wilmington, Del., in 1638. Tradition states that when, in 1698, they built a church at Wicaco (later Philadelphia) they called in an astrologer to cast a horoscope to determine the propitious time for laying

[3] There are various lists of American almanacs as may be seen in most large libraries. Upwards of 25 authors of the seventeenth century are included.

the foundation stone. The sundial said to have been used by him, the work of Christopher Schüssler (Augsburg, 1578), is now owned by the American Philosophical Society. On its base is engraved a planisphere.[4] If such an instrument, which in this case a photograph shows to have been exceptionally well made, could have been used by a mere astrologer, others of good workmanship were probably known elsewhere in the colonies.

It is interesting to observe that the first mathematicians in the New World looked upon mathematics as merely an aid to the study of astronomy, as their European predecessors had so often looked upon the latter as the handmaid of astrology. This tended to make the mathematician astronomically-minded, and this was largely the case until the latter part of the nineteenth century. It had good precedents in the work of Newton in England, Laplace in France, and Gauss in Germany, and the result is seen in that of Benjamin Peirce and Simon Newcomb in this country.

Thus the seventeenth century in America produced no mathematics worth the name. The settlers were more concerned with combatting the hardships of life in a new country, with religious quarrels, with fighting the bogy of witchcraft, with petty political feuds, with the small gossip of small communities, and with devising tortures (mental and physical) for those whom they hated, than they were with the masterpieces of contemporary European literature and science. The century that saw the work of Galileo, Kepler, Gilbert, Napier, Fermat, Descartes, Pascal, Huygens, Newton, and

[4] H. D. Paxson, *Where Pennsylvania History Begins* (cover title), Philadelphia, 1926, p. 46.

Leibniz, in countries from which the settlers had come, saw among the intelligentsia no apparent appreciation of the discoveries of scholars of this class. Indeed, even the leaders seemed to sanction the very European religious persecution from which so many had fled, and actually to attempt to produce such erudite bigots as Cotton Mather, who should have known the writings of these world leaders, but who wasted his abilities upon "daemons and witchcraft," "evil spirits," "evil angels," and other evidences of disordered minds that today pass the understanding of even the most reactionary.[5]

[5] See Mather's *The Wonders of the Invisible World*, Boston, 1693.

# CHAPTER II

## THE EIGHTEENTH CENTURY

### 1. GENERAL SURVEY

The eighteenth century was the first to show much interest in America in any phase of mathematics. This interest is not, however, seen in the records of the preparatory schools, and appears only slightly in those of the colleges. In spite of what Judge Thomas Holme wrote in 1696 about the ability of the schoolmasters of Pennsylvania to bring pupils to school

> And to instruct and make them quick
> In all sorts of arithmetick,

the subject was poorly taught. In Massachusetts the standard was even lower than in the preceding century when every town of a hundred families had a grammar school, whereas in 1789 there was only one for every town of two hundred, thus closing the facility for college preparation to one hundred twenty Massachusetts towns.[1] Although this condition, as it related to mathematics, concerned only arithmetic, it had the indirect result that relatively fewer boys were prepared for entering a college where they might have had some mathematical training however slight. Indeed, a large number of people opposed the idea of public education on the ground that it made boys lazy and dissatisfied with farm life, and led to religious skepticism. A similar

[1] A. M. Earle, *Child Life in Colonial Days*, New York, 1929, p. 70, seq.

argument was common until the latter part of the nineteenth century in Europe as well as in America.

The redeeming feature of the situation lay in the fact that mathematics in the eighteenth century, as we have seen in earlier periods, did not originate generally in the schools, nor did it spread chiefly through this agency. If we except such mechanical features as the elementary operations of arithmetic and algebra and consider the progress of real mathematics, neither the elementary schools of this country nor the colleges were much concerned in that period with the subject. In Europe this had, as we have already seen, frequently been the case in the seventeenth century, as witness further the epoch-making discoveries of Fermat, Pascal, Desargues, Leibniz, Napier, and Harriot, although we must recall at the same time the work done in Cambridge by men like Barrow and Newton, and at Oxford, with more erudition but less originality, by Wallis.

In classical times it was men of the Maecenas type who fostered publications; later it was the Church which gave opportunity for study and for the spread of knowledge through the scribes in the monastic centers; with the advent of printing it was the kings and nobles who, partly in payment for florid dedications, made it possible to publish treatises on mathematics as well as those on letters; still later there came into being learned societies, fostered by the Church or the State, and these provided media for making known the discoveries and inventions of their members; and finally, so general was education becoming, it was through commercial publishers of texts, of periodicals, and of meritorious but somewhat popular books upon the various sciences that

certain mathematical discoveries became matters of a somewhat general interest, as is the case in American communities today.

As to the eighteenth century, America could not depend, for the encouragement of mathematics, upon support from the churches, from the State, from nobles seeking notoriety through dedications of learned treatises, or through any extended popular support. There were no native periodicals devoted to science nor were there any learned societies until about the middle of the century. The colleges showed no originality in mathematics, the Church had but little to offer in this field, and but few individuals produced anything worth mentioning.

As to the closing years of the century it may be said, in the words of President Barnard of Columbia University, that "in any review of the progress of science, . . . the period which lies between the declaration of independence and the close of the eighteenth century may, without danger of any important omission, be passed over in silence."[2]

In considering the mathematics of the 18th century we must rely chiefly upon reports concerning (1) a small number of colleges, most of which later became universities; (2) the learned societies, together with their publications; (3) articles by Americans, whether published in this country or in Europe; (4) the biographies of men of learning who contributed directly or indirectly to an appreciation of the subject, even if they did little to advance it.

[2] F. A. P. Barnard (1809–1889), president 1864–1889. *The First Century of the Republic*, New York, 1876, p. 294.

## 2. THE COLLEGES

We shall first consider typical cases showing the work done in certain of the nine colonial colleges existing in America in 1776, with the present names of the colleges or universities, and the approximate dates of their foundation: 1. Harvard, 1636; 2. William and Mary, 1693; 3. Yale, 1701; 4. Princeton, 1746; 5. Pennsylvania, 1751; 6. Columbia, 1754; 7. Brown, 1764; 8. Rutgers, 1766; 9. Dartmouth, 1770.

To these might be added Washington and Lee which developed from an academy founded at Lexington, Va., in 1749, becoming a college in 1813, and a university with its present name in 1871.

As a preliminary to our consideration of such topics as the preparation for college, the curricula, the faculties, the facilities for study, and the union of mathematics with astronomy and physics, it is desirable to call attention again to the influence of the Church.

The hope of the Puritans that a Holy Commonwealth might arise in America, and the religious zeal of men like Cotton Mather, Jonathan Edwards, George Whitefield, and John Woolman, naturally had their influence upon the early colleges. With a similar purpose Samuel Johnson, first president of King's College, from which developed Columbia University, and Dean (later Bishop) Berkeley[1] who exposed the greatest weakness of the Newtonian theory of fluxions and whose *Collected Works* treat quite extensively of mathematics, tried in vain to turn the mass of people in the colonies back to a recog-

[1] 1685–1753. He came to Newport, R. I., in 1728 and remained three years.

nition of the Anglican Church. These efforts had a strong influence upon the education given in the colleges which had multiplied rapidly by the middle of the nineteenth century. This accounts for the fact that, up to about 1850, such a considerable number of professors of mathematics were clergymen whose chief interests were in theology rather than in the subject they were supposed to teach. This was entirely natural, since the colleges, following the original purposes of those of Great Britain and Ireland, were established, as already stated, chiefly for the training of the clergy. Thus we find among the professors of mathematics given in a list of 1833 such doctors of divinity and eminent clergymen as Joel S. Bacon, Homer J. Clark, Jeremiah Day, George W. Eaton, Hosea Hildreth, Henry Ruffner, J. M. Sturtevant, Silas Totten, William Wall, and W. P. Aldrich. Indeed, in this list of professors and instructors in mathematics one-sixth were recorded as trained for the church.

The catholic missionaries, particularly the Jesuits, seem to have done quite as much as the protestants in the establishing of schools. Thus a Jesuit college was established in Kaskaskia (Illinois) in 1721 and was maintained under a French charter until the banishment of the Jesuits in 1765. The name "college," however, seems to have referred to a residential center of church activity rather than to a school of any size, the work being almost wholly theological. In Louisiana the Spanish authorities did little for education, and under the French supremacy the schools were almost wholly private.

The preparation for entering the meager mathe-

matical courses in college was pitifully weak. Even as late as the middle of the 18th century there were only one or two academies in all New England that pretended to prepare pupils adequately for college, extra tutoring being necessary, usually given by some local clergyman.

The nature of the mathematics taught at Harvard in the first half of the eighteenth century is fairly well known.[2] For example, we know that **Isaac Greenwood**[3] studied for a time in England and was Hollis Professor of Mathematics from 1728 to 1738. He taught algebra of a formal type as shown by two extant manuscripts, one written about 1738 by Samuel Langdon,[4] and the other by James Diman[5] written in 1730–31, the year of his graduation. The two manuscripts are quite alike and represent the work given by Greenwood. They are evidently based upon the treatise of John Wallis[6] as shown by their similarity to portions of this work.[7]

The course in algebra included the usual operations, which were developed, quite beyond their mathematical importance, by the English algebraists of the 17th century; various methods of solving "adfected Quadratic Equations," also carried beyond any practical value; "The Resolution of Cubick Equations," "The Method

[2] Lao G. Simons, *Introduction of Algebra into American Schools in the Eighteenth Century*. Washington, 1924—the source of much information in this chapter.

[3] 1702–1745; Harvard A.B., 1721; A.M., 1724.

[4] 1723–1797; Harvard A.B., 1740; A.M., 1743; president of Harvard, 1774.

[5] Born 1707; A.B., 1730; A.M., 1733; librarian of Harvard, 1735–1737.

[6] 1616–1703; Savilian professor of geometry at Oxford, 1649–1703.

[7] His *Arithmetica Infinitorum* was published at Oxford in 1655, and his *Opera Mathematica* in 1693–1695.

of Converging Series," and "Dr. Halley's Theorems for Solving Equations of all sorts."

Besides the work in algebra there is, in each manuscript, a "Method of resolving Geometrical Problems algebraically," these being twenty-four in number, with diagrams, and the algebraic treatment, usually involving quadratics, being set forth in full.

The interest in these manuscripts lies chiefly, however, in the references which they make to British writers. For example, each makes mention of Oughtred (1574–1660), Raphson (d. before 1715), and Halley (1656–1742); and Diman gives a list of "Books perused in *y*[e] review of my Algebra made in 1738," the last year of Greenwood's professorship: "1. Harris Lexicon Technicum.[8] 2. Chambers Cyclopaedia.[9] 3. Wolfius Elementa matheseos univers."[10]

Each of these manuscripts refers also to the work of Euclid, Apollonius, and Archimedes, and Langdon speaks of a rule for the expansion of a binomial "given by S[r]. Isaac Newton, see p. 139." This page is in a work by John Kersey (1616–1677), *The Elements of that Mathematical Art commonly called Algebra*, 1st edition, 1673.

Thus we have direct information concerning the European works which influenced Greenwood, the first important college professor of mathematics (as distinct from related sciences) in America, and the author of the

[8] John Harris (c. 1667–1719), *Lexicon technicum, or an universal English Dictionary of Arts and Sciences*, 1704; particularly strong in the articles relating to mathematics, physics, and astronomy.

[9] It first appeared in two volumes in 1728.

[10] Christian von Wolf (1679–1754), *Elementa matheseos universae*, 4 vols., Halle, 1733.

first arithmetic published (1729) by an American.[11] Hodder's English arithmetic had already been reprinted (25th edition) in Boston in 1719, however, by James Franklin, the uncle of Benjamin, in whose printing office the latter had served as an apprentice. Greenwood undoubtedly became familiar with various standard works while in England, and so far as we know, he was the first to introduce those above the arithmetic stage to college students in this country. He seems to have made a very good selection, chiefly from English works, although with our present knowledge of the field many additions would seem desirable.

A little light upon the course offered even earlier at Harvard is afforded by a student's notebook of 1718–19, representing the work of his freshman or sophomore year. It contains sections on arithmetic and geometry, but is not as advanced as the two manuscripts above mentioned.

As to the work of this period we have further information in the principles officially laid down in 1726 to guide Greenwood in his courses at Harvard. These require "That the Professor be a Master of Arts and well acquainted with the several parts of the Mathematics & Experimental Philosophy," and that he give instruction in "Natural Philosophy & a course of Experimental in which to be comprehended, Pneumaticks, Hydrostaticks, Mechanicks, Staticks, Opticks, &c in the Elements of Geometry to-gether with the doctrine of Proportions the Principles of Algebra Conic sections, plain & Sper-

[11] On this point see Lao G.Simons, *loc. cit.*, p. 68, the results of whose painstaking research have been freely used in the preceding pages and later. See also *Scripta Mathematica*, I, 262, New York, 1932.

ical Trigonometry with the general principles of Mensuration, Plain & Solids, in the Principles of Astronomy & Geometry, viz, the Doctrine of the Spheres the use of the Globes, the motions of the Heavenly Bodies according to the different Hypotheses of Ptolomy Tycho Brahe & Copernicus with the general Principles of Dialling," etc.[12]

A report in 1726 states that "the senior sophisters, besides Arithmetic, recite Allsted's geometry, Gassendus's astronomy in the morning," so the above information appears reasonable. The program, however, seems rather pretentious, and in any case it shows again that the ultimate aim of mathematical work in the colleges was astronomy. Dialling held a position which had been inherited from early times when the sun-dial was the chief instrument for the measurement of time.

Another source of information as to the work done at Harvard in the eighteenth century is seen in the bachelors' theses proposed for discussion at the commencement exercises. These theses, stated in printed form, are known as far back as 1653, and those of 1708 show little improvement. In 1711 however, some simple statements relating to conics are found, and as early as 1719 there is evidence that Newtonian fluxions had gained admission to an American college; for example, "Fluxio ex quantitate fluente Invenitur." This was about thirty-seven years before John Winthrop taught the subject there. There is also evidence, in the same year, that the theory of music was considered a part of mathematics,

[12] *Harvard College Papers*, I, Jan. 18, 1726, in the Harvard Library; quoted by Lao G. Simons, *loc. cit.*, p. 45. In all quotations the spelling, construction, punctuation, and the like have been closely followed.

as was the case from classical times through the Middle Ages, the proposition to be disputed being, "Dias et Trias harmonica sunt fundamenta contrapuncti musici." In 1721 Harvard included algebra among its theses, but the propositions for debate related merely to the general nature of the subject. It is interesting, however, to observe that a case similar to Fermat's theorem appears among the theses set by Yale (1720), Harvard (1731 and 1780), and Brown (1773)—"Duo numeri biquadrati summatim sumpti numerum quadratum constituere non possunt" ($x^4$ plus $y^4$ cannot equal $z^2$).[13]

The work done at Harvard under John Winthrop, the best equipped American of his time in astronomy and mathematics, is considered later.

Next to Harvard in the order of seniority is William and Mary, and we possess some reliable knowledge of the work done there in mathematics as early as 1711. In speaking of Harvard, reference was made to Greenwood as the first "important" college professor of mathematics in America. The limitation was due to the fact that there were two earlier professors at William and Mary. The first was a man named Le Fevre, appointed in 1711 and dismissed in 1712 as "negligent in all the post of duty and guilty of some other very great irregularities," and the second was the Reverend Hugh Jones, appointed in 1717. The latter was well trained in an English university, but unfortunately remained in office only five years. He later speaks in his *The Present State of Virginia* (London, 1724) of having written an

[13] For lists of these theses in the early colleges and the methods of procedure at commencement see Lao G. Simons, *loc. cit.*, p. 30.

*Accidence to the Mathematicke* in which he included "Arithmetick," Algebra, Geometry, Surveying of Land, and Navigation, but no copy of such work is known to be extant, and it seems never to have been published. The statement goes to show, however, that the purpose of the course at William and Mary was to lead to surveying and navigation.

It could have been no ordinary faculty that trained Thomas Jefferson, then a youth of twenty, to leave this college with a good working knowledge of Latin, Greek, and French, with a taste for mathematics which he was to cultivate still more in later years, with a knowledge of the sciences which led him into correspondence and familiar intercourse with the leading scholars of France and England, and with an appreciation of languages which enabled him to acquire the power of conversing in Italian and Spanish and of reading Anglo-Saxon. Recognizing as we must that such ability comes as a gift of nature, never-the-less the College of William and Mary doubtless stimulated his inherited powers to greater efficiency than would result from courses taken in the mediocre schools of his time.[14] His influence upon mathematical studies in general is considered later in this chapter.

In the early years at Yale, when the college was merely a "Collegiate School" at Killingworth, having its beginning in the parsonage of the Reverend Abraham Pierson who died in 1707, the mathematics taught to a freshman was, as already stated, hardly more advanced than that of the seventh grade of a modern

[14] See "Thomas Jefferson and Mathematics," by D. E. Smith, in *Scripta Mathematica*, I, 3–14, 87–90.

elementary school. The rector taught physics and mathematics to the seniors, reading from his own notes and making no use of textbooks. Indeed, it is said that there was but one copy of any of Newton's works in all New England at this time. The only encouragement to the study of mathematics in the "Collegiate School," so far as we know, was possibly given by Noadiah Russell, a member of the board of trustees, who was a maker of almanacs. President Woolsey states that the first mathematical work of which he found any evidence in Yale was Ward's *Mathematics* which contained "a meager collection of the most elementary propositions in geometry and in conic sections."

The work at Yale in the latter part of the first quarter of the century was of about the same character as that done at Harvard. Thus the records show that courses in higher plane curves were given at Harvard in 1735 and at Yale in 1739, and that the subject of fluxions was studied at Harvard in 1751 and at Yale in 1758. These dates must not, however, be taken as representing the earliest appearance of these courses, since we have seen that some work in fluxions was given at Harvard as early as 1719. From the commencement theses offered after the school had become Yale College (1718) we know that the course included arithmetic of a very elementary nature, algebra with some slight knowledge of equations, trigonometry as far as the solution of plane triangles with the aid of logarithmic functions, and an introduction to the measurement of the sphere, with a knowledge of how to set a sundial and of the method of locating a star. As would be expected, the purpose of the course, aside from a somewhat exaggerated dis-

ciplinary value, was to give the student such a knowledge of astronomy as would help the navigator and would lead to the location of a star by an amateur astronomer.

As to the effect of such a course upon the student we have a note written by Samuel Johnson already mentioned as the first president of Columbia. He was a graduate of Yale in the class of 1714 and was a tutor in mathematics there from 1715 to 1719, being assisted part of the time by the minister of the village. This note, written in 1714, is as follows: "Ho when I was at Coledg I was taught nothing but to be a conceited Coxcomb like those that taught me. Indeed we had no Books & our Ignorance made us think we needed none."[15]

A somewhat similarly frank statement with respect to the work at Yale more than a half century later is due to the poet John Trumbull (1750–1831) who was graduated there in 1767 and was a tutor from 1771 to 1773 and whose satirical poem "The Progress of Dulness" (1772–1773) was a well-justified attack on the current education, although not on mathematics in particular.

The greatest inspiration given to the work at Yale in the midde of the century was due to Thomas Clap.[16] He was probably the first to publish a catalogue of an American college library (New London, 1743) and was a man of broad culture. Among his publications was one on the nature and motions of meteors. In his plan

[15] These and other notes of Johnson's are now in the Columbia University Library.

[16] 1703–1767; he became Rector (president) of the college in 1739 and drew up the charter which was granted in 1745 and which legalized the name "Yale College."

of studies he included arithmetic and algebra (first year), geometry (second year), and "mathematics and natural philosophy" (third year). In 1766 he wrote a history of Yale and showed how the work had developed so as to include, for those who were prepared for such studies, navigation, the calculation of eclipses, conic sections, and (as we have seen) fluxions. In 1760, and towards the close of his administration, Clap wrote that many of the juniors understood surveying, navigation, and the calculation of eclipses, and some were fairly proficient in conic sections and fluxions. On the scale of mathematical ability this was not much beyond the freshman or first half of the sophomore year at present. It reflects, however, the demand for practical mathematics created by the opening of new territory for surveying, and the developing of the East India trade with the resulting need for increased skill in navigation. In 1770 a new professorship in mathematics and natural philosophy was established and Nehemiah Strong was called to the chair. It is said that he carried his mathematical class "as far as any of them would go in the Principia of Newton;" but President Woolsey adds the statement that this must have been "a very rare thing." The nature of the Yale theses for the bachelor's degree may be inferred from a few taken from the earliest extant broadside list, that of 1718, the numbers and statements being as there given:

5. Multiplication by a decimal fraction decreases the value of any number; division increases it.[17]

8. Algebraic number gives neither greatest nor least.

[17] The case of a proper common fraction seems not to have been considered as analogous.

13. What is involved in involution is resolved by evolution.

16. Primary logarithms are formed by the repeated extraction of the square root.

19. Given the base and altitude, the angle at the base can not be found by the use of a line of sines.

21. Trigonometric problems can be solved most accurately by the use of logarithms.

25. The surface of a sphere is four times the area of its largest circle.

26. The right ascension of a star is its meridian distance from the beginning of the Ram[18] numbered by degrees on the equator, the longitude on the ecliptic.

29. The angle at the base of the horizontal sundial must agree with the elevation of the pole.

As to the college curriculum throughout the eighteenth century, there was no significant improvement upon the work offered at Harvard and Yale in the first two or three decades. As regards Princeton, for example, the journal and some letters of Philip Vickers Fithian, who was a student there in 1770–1772, have been edited by John Rogers Williams (Princeton, 1900), and these show the work then being done. It appears that Euclid and "Fluctions" were taught, and that use was made of Saunderson's *Algebra*[19] and John Hill's arithmetic, which included ninety-nine "Problems and Questions in algebra," a work published in London in 1712 and several times reprinted

Fithian had studied under William Churchill Houston, who was appointed professor of mathematics and natural philosophy in 1771. What this meant in a pecuniary sense, may be judged by the salary of £150 a year paid to Hugh Williamson as professor of mathe-

[18] Aries, the first zodiacal constellation, the vernal equinox.

[19] London, 1756; 2d ed., here used, 1761.

matics at Princeton in 1767, due allowance being made for the difference in the purchasing value of a pound then and now.

Some advance in the standard of instruction was probably made there after the appointment in 1788 of Walter Minto (1753–1796) as professor of mathematics and natural philosophy. This appointment was made in part on the recommendation of the Earl of Buchan, whose letter is still preserved in the University and which speaks of him as "a gentleman likely to prove a very useful accession to the mathematical department of America." Minto had collaborated with the Earl in a work on Napier (Perth, 1787), which contains a sketch of the early history of calculating machines. He also made a translation of the *Cours de mathématiques* (Paris, 1782) of Charles Bossut (1730–1814), but it seems never to have been published although the manuscript is in the University library. It shows the nature of the work in the college and the influence of French mathematics there. There was more reason for France to be favored at Princeton after the Revolutionary War than was the case in most other educational centers, for it seems to have suffered considerably during the conflict. Addressing the trustees in his inaugural oration, Minto remarked on the necessity of "replacing such parts of the philosophical[20] apparatus, as we were deprived of by the incidents of the war, which was prosecuted by our enemies in the true spirit of the Goths and Vandals." In the spirit of fairness, however, he later stated that "the American soldiers had their share in the depredations."

[20] *I.e.*, physical and astronomical.

Before coming to Princeton, Minto, who had studied at the University of Pisa, made a contribution to mathematical astronomy by solving the problem "To determine the circular orbit of a planet by two observations" (London, 1783). His interests in his new position were rather with astronomy than with mathematics. It may very likely be due in part to his advocacy of the policy that, through his professorship and into the first decade of the new century, the junior and senior classes read no Greek or Latin but devoted their time to mathematics, literature, and other subjects.

Our knowledge of the work done in the eighteenth century at the University of Pennsylvania (then known as the University of Philadelphia) is enriched by a notebook of one of the students, Samuel Miller, written in 1788. It shows that Robert Patterson (1743–1824) then professor of mathematics, gave courses in "Oblique Spheric Trigonometry," stereographic projection of the sphere, conic sections, surveying, navigation, spherical geometry, and fluxions for rectifying a curve. Patterson was born in Ireland of Scottish parents, and although not a university graduate when he came to America, he was a teacher of unusual ability. He entered the University of Pennsylvania in 1779 and later became professor of mathematics and vice provost. He was active in the American Philosophical Society, and was its president from 1819 to 1824. He published an arithmetic (1818) and an astronomy (1808), and left the manuscripts of two other works. He also wrote articles for the *Proceedings* and the *Transactions* of the American Philosophical Society, but they, like his books, have value only as showing the low state of mathematics

at that time. Further evidence of this neglect is seen in a set of three notebooks of Robert Brooke of the class of 1793, but not a graduate, which books are still extant.

As to the work done in the colleges of the British colonies in the third quarter of the eighteenth century, and in particular in New York and Pennsylvania, the following general statement has been made in an English history:

"Sir James Jay, collecting funds in England for King's College, New York, and Dr. William Smith, who was also collecting for his college in Philadelphia, drew for the King, the nation and the Archbishop of Canterbury, a pitiful contrast between the intellectual culture in the newly acquired Canada and the uncultured backwardness of the older English colonies."[21]

Whether or not the comparison was just, it is certain that the work in the colleges of the new republic, judged by their curricula, was of a low grade. For example, in *The Statutes of Columbia College in the City of New York*, printed in 1785, "The Plan of Education" required only that, for entrance, the candidate "shall understand the four first rules of Arithmetic, with the rule of three," in the field of mathematics. In the "Freshmen Class" the student devoted three class periods a week to "Vulgar and decimal Fractions, Extracting the Roots, Algebra as far as quadratic Equations." In the Sophomore year the class in mathematics met once a day and studied "Euclid's Elements. Plain and Spherical Trigonometry. Conic sections and

[21] Thomas Hughes, *History of the Society of Jesus in North America*, London, 1917

The higher branches of Algebra." No further mathematics was required except such as was taught incidentally in the courses in physics and astronomy in the junior year.

### 3. PRIVATE INSTRUCTION

The paucity of instruction in mathematics in the colleges is also apparent from the fact that it was necessary to organize private classes for the purpose of giving instruction in subjects which should have had place in the curriculum. This is shown by the advertisements of teachers in the newspapers of the eighteenth century.[1] From these it appears that in 1709 Owen Harris gave private instruction in Boston in "Plain and Sphaerical Surveying, Dialling, Gauging, Navigation, Astronomy," and other topics. In 1727 Isaac Greenwood did the same, his offering including such items as "Sir Isaac Newton's incomparable Method of Fluxions, or the Differential Calculus, to-gether with any of the Universal Methods of Investigation used by the Moderns; the Elements of Euclid and Appollonius," and "the Modern Discoveries in Astronomy and Philosophy." This is particularly interesting because it seems to show a knowledge of the Leibnizian as well as the Newtonian calculus. In 1739, after his dismissal from Harvard, a similar advertisement inserted by him includes "Navigation, Surveying, Gauging, . . . Mechanics, Opticks, Astronomy, &c." In 1743 Nathan Prince, for thirteen years a tutor at Harvard, advertised in Boston that he

[1] This has been considered by Lao G. Simons, who examined systematically the files of eight of the leading papers of the period. The details appear in her work already cited, pp. 67 seq.

proposed to "open a school in this Town for the instructing young Gentlemen in Geometry and Algebra; in Trigonometry and Navigation; in Geography and Astronomy, . . . Surveying, Gauging, and Dialing; and in the General Rules of Fortification and Gunnery." Similar advertisements appeared in New York, inserted by James Lyne (1730), Alexander Malcolm (1732), and John Wilson (1749), besides notices referring chiefly to arithmetic. In Philadelphia, Theophilus Grew (1734), Alexander Buller (1741), and John Clare (1747) also opened schools of about the same nature.

Some idea of the progress of mathematics is also seen in the advertisements of books imported for sale. These include announcements of works by Kersey (1729), Maclaurin (1771), Newton (1751), Saunderson (1777), Simpson (1772), Sturm (1729), and Wolf (1748), these dates referring to the advertisements.[2]

## 4. EQUIPMENT FOR STUDY

The equipment of the colleges was very limited. It consisted of pitifully small libraries, of a limited number of poor textbooks, and of a few astronomical instruments for use chiefly in the study of navigation or in the observation of eclipses.

As an example of the library facilities in the best colleges it may be noted that Harvard led the way in this country. In 1638 the Reverend John Harvard (1607–1638), a graduate M.A. of Emmanuel College, Cambridge, died and left 260 books and £780 to the "schoale or colledge" which had been founded at "Newtowne" by the General Court of the colony in 1636. In honor of

[2] Simons, *loc. cit.*, p. 72.

his gifts the college was named (1639) Harvard College. The books were probably for the most part of a religious nature, but in spite of this fact Harvard was the first to show any contact with European treatises of any importance in the field of mathematics and astronomy. The first noteworthy scientific treatise used there, naturally as a library reference book, was Gassendi's *Institutio astronomica* (Paris, 1647).[1]

The Harvard Library, in the first quarter of the eighteenth century, contained, however, a carefully selected lot of classical works in the fields of mathematics, astronomy, and physics. From the *Catalogus Librorum Bibliothecae Collegij Harvardini* (Boston, 1723) it appears that there were upwards of 150 books of this kind in the library at the time of its publication. These included editions of Aristotle, Apollonius, Hayes, Harriot, Huygens, Kircher, Diophantus, Euclid, Gregory, Gassendi, Munster, Ptolemy, Pappus, Kepler, Vincentius, Vieta, Wing, Wallis, Boyle, Tycho Brahe, Barrow, Branker, Descartes, Galileo, Hawksbee, Leybourn, Newton, van Schooten, Alsted, Napier, Oughtred, Peurbach, Recorde, Snell, Whiston, and various others. The list was prepared for Thomas Hollis in order that he might purchase additional books for the library, and many valuable works must have been secured before the fire of 1764 which consumed the entire collection. The *Massachusetts Gazette* of February 2, 1764, referring to this fire, which occurred on the night

[1] Petrus Gassendus (Pierre Gassendi), 1592–1655. See F. E. Brasch in the *Scientific Monthly*, XXXIII, 336, 448. There were later editions. To Mr. Brasch we are also indebted for information referring to the list in the following paragraph.

of January 24, spoke of the library as "the repository of our most valuable treasures, the public library and philosophical apparatus." It contained at that time upwards of 5000 volumes, and the apparatus included the material given by Hollis.

The first years at Yale were naturally more favorable from the mathematical standpoint than those at Harvard, for a large literature had come into being since the latter was founded. It was due chiefly to the efforts of Jeremiah Dummer, the London agent for the Connecticut colony, that a worthy beginning was made. He secured a gift of nearly a thousand volumes and sent them to the Collegiate School at Killingworth. There being no place for them there, in 1716 the trustees moved the school to New Haven where a gift of books and of saleable goods sent from India by Elihu Yale was also added to the store and led to the naming of Yale College (1718, legalized in 1745). Among the gifts were two from Newton—the second edition (1713) of the *Principia* and a copy of his *Optice* (1706), both still in the Yale Library. Halley also showed his interest in the new college by sending a copy of his *Apollonius*. It was the opportunity for studying such books as these that kindled the ambition of Samuel Johnson, already mentioned as the future president of Columbia, to acquire a working knowledge of mathematics sufficient for their mastery. This was a new revelation to him, for he had been graduated with a high opinion of his own mathematical ability and had found in reading Newton's works that his college training in this field was almost worthless. He speaks of himself as "suddenly emerging out of the glimmer of twilight into the full sunshine of

the open day." Moreover the effect upon the college authorities was salutary, for the Rector, the Reverend Timothy Cutler, added (1719) to the library two books quite essential to a better understanding of Newton —Alsted's Geometry (probably the *Elementale mathematicum* . . . II. *Geometria*, Frankfort, 1611) and Pierre Gassendi's *Institutio Astronomica* already mentioned. In the catalogue of the Yale library, printed in 1743, it appears that there were 55 books listed under Mathematics, seven being on astronomy. Among the standard works were "Des Cartes' Geometria," "Newton's Principia, Optics," and "Optical lectures," and various works by Euclid, Apollonius, Theodosius, Ozanam, Wallis, Bion, "Hayes of Fluctions," Vieta, Taylor, and Cotes —a fairly good collection for the time.

Such were the rather meager library facilities offered to a student of mathematics in the leading American colleges in the first part of the eighteenth century.

## 5. TEXTBOOKS

As to textbooks, only a few were printed in America in the eighteenth century, and these were principally elementary arithmetics. As already stated, James Hodder's English arithmetic was reprinted in Boston in 1719, "The Five and Twentieth Edition," revised by "Henry Mose, late servant and successor to the Author." The revision by Mose appeared in the eighteenth English edition (1693) and the twenty-fifth edition came out in 1714, the Boston edition of 1719 being a reprint of the latter. It contained nothing which related to the colonies, but it states that there are "above a Thousand Faults Amended," showing the general unreliability of

the original work. This was not the first mathematical work printed in the New World, for the *Sumario Compendioso* of Brother Juan Diez[1] appeared from the press of Juan Pablos, in Mexico, in 1556.[2] This was the first textbook of any kind, except for religious instruction, to be published in the New World. It does not, however, relate to the progress of mathematics in that part of America with which we are concerned.

Although Hodder's was the first arithmetic printed in English in the colonies, other books containing chapters on the subject had appeared. In William Bradford's *The Secretary's Guide, or Young Mans Companion*, which was published by the author in New York (1728), the prefatory letter "To the Reader" states that "It is now above thirty years since I first compiled this short Manuel, during which time several Impressions have sold off . . . and now this fourth Edition . . . ." The original edition must, therefore, have appeared before 1698. In this fourth edition pages 31–73 relate to "Arithmetick made Easie." In the fifth edition (1738) the arithmetic portion fills seventy-two pages.

There is also one other book preceding Hodder which may be mentioned—*The Young Secretary's Guide: or a speedy help to Learning* (Boston, 1718) a reprint of an English work which had already gone through four editions and which contained a few pages on arithmetic as used in legal documents.

The first arithmetic written by an American and de-

[1] *Sumario cõpẽdioso . . . Fecho por Juan Diez, freyle.* The mathematical portion was translated and edited by D. E. Smith, Boston, 1921.

[2] "El qual fue impresso en la muy grande y muy leal ciudad de Mexico, en casa de Juan Pablos Bressano."

voted wholly to this subject was published anonymously at Boston in 1729. Its authorship was in doubt until it was established, by means of a contemporary advertisement in a newspaper, to have been the work of Isaac Greenwood, first Hollis Professor of Mathematics at Harvard.[3] It is noteworthy because its treatment is more advanced than that given by Hodder, and it omits much that was already obsolete. The following extracts from his "Advertisement" shows the modern spirit of the author:

> He has had his Mind all along upon Persons of some Education and Curiosity. . . .
>
> He has thought it improper to go into an elaborate Explanation of the Rules in the lower Parts of Arithmetic, as most Authors have done, seldom to any other Effect than to perplex the Subject, and excite a secret Prejudice. . . .
>
> He has confined Himself to such Rules as are of the greatest Repute with the Modern Mathematicians, and what they make Use of in their Works. . . . And this He thinks a sufficient excuse for Omitting several Sorts of Division,[4] the different Methods of extracting the Roots, and some Particular Ways in the Rule of Practice. . . .
>
> There are many things in the following Treatise of greater Curiosity, than Necessity in the Practice of Numbers. Such are the Methods . . . of contracting Decimal Multiplication and Division, the Rules concerning Circulating Figures, Sir Isaac Newton's Contraction in the Evolution of the Square Root, &c. . . .

The purpose of this list of quotations from an arithmetic is to show that here, at last, was an author who

[3] This advertisement was discovered by Lao G. Simons and made known in her work already cited.

[4] This refers to such methods as *per colona*, *per toletta*, *per repiego*, *per il scapezo*, *per galea*, and *divisio ferrea*, carried over from Italian writers to northern Europe and the British Islands, some of them finding place in books printed in England.

was not afraid to take a decided step in advance, to show some mathematical ability, and to give evidence of having read the works of mathematicians. But, like Bishop Tonstall, who wrote the first scholarly arithmetic in England, some two centuries earlier,[5] Greenwood reached a relatively small audience. There was no second edition of the latter's work, although Tonstall was more successful in his venture.

Another evidence of scholarship showed itself in a Dutch arithmetic (New York, 1730) by "Pieter Venema, Mr. in de Mathesis en Schryf-Konst," which contained a little work in algebra. Venema had already published an algebra in Groningen[6] and had at one time been a pupil of Jean (I) Bernoulli's, who was a professor at Groningen from 1695 to 1705. It is probable that he was as good a mathematician as any of the early teachers in the colonies. As to the next step beyond arithmetic the most popular book seems to have been a work by John Ward of Chester, *The Young Mathematician's Guide*, which was published in London in 1709, a work of scientific merit and evidently considered to be more teachable than any of its rivals. It was in use in Harvard as early as 1726,[7] and probably earlier, and as late as 1794. From a note by Samuel Johnson it seems to have been used in Yale even earlier than in Harvard. It is also referred to in documents relating to the Friends' school in Philadelphia about the middle of the century. It included some work on algebra, Euclidean geometry, conic sections, "The Arithmetic of Infinites," and gaug-

[5] *De arte supputandi*, London, 1522.
[6] In 1714, with several later editions.
[7] The third edition, with corrections, was printed in London in 1719.

ing. Besides this work, Nathaniel Hammond's algebra (London, 1742) and Thomas Simpson's algebra (London, 1745) were well known.

Although Greenwood's arithmetic was printed in 1729, and Venema's book in the following year, it was more than a half century before there appeared an arithmetic containing any algebra and written by a native American. This was Nicolas Pike's *A New and Complete System of Arithmetic* ("Newbury-Port," 1788), a work which became very popular. There was no American book on algebra alone published in the 18th century, although Consider and John Sterry's *The American Youth* (Providence, 1790) contained sections on arithmetic, geometry, and algebra, and John Gough's *Pracrical Arithmetick*, a revision of a Dublin publication and containing a little algebra, was printed at Wilmington in 1798 (2d ed., 1800).

The German influence on pure mathematics was slight in the eighteenth century. Christoph Saur, a native of Westphalia, established a press in Germantown (Germanton), Pennsylvania, and printed the first newspaper, in German type, in the colonies. He printed also the first German bible (1743) and numerous textbooks. He published two ready reckoners, one (1774) being a German translation of the seventh edition of Daniel Fenning's popular tables.[8]

To observational astronomy, however, the Germans

[8] The subject of American arithmetics has been so fully discussed in Professor Karpinski's *History of Arithmetic* (Chicago–New York, 1925), as to render any further treatment unnecessary at this time.

The subject of the German influence has been discussed by G. von Bosse, *Das Deutsche Element in den Vereinigten Staaten*, Stuttgart, 1903, p. 41.

made several contributions. An example of the work they did is seen in Daniel Frühauf's *Beschreibung der bevorstehenden Partial Monds-Finsterniss* of April 7, 1773, (Philadelphia, 1773), a pamphlet of sixteen pages quite comparable to any similar European publication.

### 6. ASTRONOMY, NAVIGATION, AND GEODESY

In considering the equipment of the colleges it remains to mention more at length the instruments used in surveying, astronomy, and physics. As to the first of these, there must have been a good supply of ordinary field instruments of a fair degree of precision, not only in colleges but in the possession of the surveyors themselves. This circumstance requires a brief consideration of the art itself.[1]

As in the seventeenth century, so in the eighteenth, the subject of surveying and map-making was the chief application of elementary mathematics in the colonies, excepting simple computation and the understanding of the calendar. The newly-cleared land had to be tilled and the forests had to supply timber and fuel. These needs rendered the office of surveyor-general a necessity, with local surveyors in every community of any size. Important landowners like Jefferson and Washington generally had the instruments and knowledge essential for their purposes, and numerous surveys made by Washington, in the days when this was his calling, are still extant—for example in the Library of Cornell University.

[1] For a bibliography of the subject see J. H. Gore, *A Bibliography of Geodesy* in the *Report for 1887* of the U. S. Coast and Geodetic Survey, Washington, pp. 311–512. Also see his *Elements of Geodesy*, 3d ed., New York, 1893, which contains a historical sketch of the subject.

There are many references in early histories to the work of the local surveyors and map-makers in this as in the preceding centuries. Such are to be found in R. G. Thwaites's, *Early Western Travels, 1748–1841.*[2] Among the prominent names given by Thwaites are those of John Melish, a Scotsman; Captain John McDowell, a Scotch-Irish settler who was killed in an Indian fight in 1742; Duncan McArthur (b. 1772), who began (1793) as a chain bearer for General Massie in his survey of Ohio and finally (1830) became governor of the state; Winthrop Sargent (1753–1820), a Harvard graduate, who was not only a surveyor but also a poet and scholar, and from 1798 to 1801 was governor of the Territory of Mississippi; Sir William Dunbar (1749–1810), a Scotsman, who settled in West Florida in 1771 and under the Spanish régime was chief surveyor and (1797) boundary commissioner, and was also a scientist of such merit as to be elected a member of the American Philosophical Society; Augustus Porter (d. 1820) who was engaged in the important surveys of the Phelps and Gorham Tract and the Holland Purchase, later (1808) becoming the first judge of Niagara County, N. Y. There was also General George Rogers Clark (1752–1818) who at nineteen years of age became a surveyor on the upper Ohio River, three years later serving under Lord Dunmore in the Indian War, after which he carried on his surveying work in Kentucky and elsewhere. He became a general in the Revolution and afterwards was Indian Commissioner. In addition to Thwaites's list there were, as mentioned later, various other prominent surveyors at

[2] Cleveland, 32 vols., 1904–1907; III, 48, 227; IV, 106, 215, 323, 325; VIII, 178; X, 70; XIII, 165; XXI, 24.

this time, including Sellers, Winthrop, Colden, Mason, and Dixon.

About the middle of the century there were several craftsmen who made, and occasionally improved upon, astronomical instruments. While these men were only indirectly connected with the development of mathematics, one name may be mentioned as representing their guild—that of Thomas Godfrey (1704–1749). Franklin wrote an obituary note (*Pennsylvania Gazette*, Dec. 19, 1749) in which he spoke of him as having "an uncommon genius for all kinds of Mathematical Learning, with which he was extremely well acquainted. He invented the new Reflecting Quadrant used in Navigation." His work, however, was almost entirely given to the making of astronomical instruments. In 1758 the *American Magazine* published James Logan's letter to Edmund Halley asserting Godfrey's claims to the invention of the Hadley Quadrant.[3] As a result the Royal Society decided that both were entitled to the honor and sent to Godfrey presents to the value of about £40.

The books on the subject printed in the colonies in this century were of no consequence. A type of the work done by the amateurs may be seen in Thomas Abel's *Subtensial Plain Trigonometry, wrought with a Sliding Rule, with Gunter's Lines . . . apply'd to Navigation and Surveying* (Philadelphia, 1761). The author was an Englishman and the work used the "subtense" of an angle (the chord of twice the angle), an ancient device, instead of the sine. He also devoted considerable space to a treatment of Aristotle's Wheel and considered the cy-

[3] John Hadley, London, 1682–1744. See A. H. Smyth, *The Philadelphia Magazines and their Contributors*, Philadelphia, 1892.

cloid as a semi-ellipse, all of which shows the rather low attainment of surveyors and authors of this type. There were, however, English works of considerable merit such as John Love's *Geodæsia*, the ninth edition of which was published in London in 1771 and was "very useful for surveyors in His Majesty's plantations in America," as the author states. It seems to have supplied a need in the pre-revolutionary period.

In this century the custom already mentioned of combining mathematics and physics was continued and the union of physics and astronomy began to be more fully recognized. Thus in Columbia the work in mathematics and physics was in the hands of men like Daniel Treadwell, who served from 1757 to 1760, a Harvard man whom John Winthrop recommended as showing "uncommon proficiency in mathematical learning, for which he appear'd to have a very good genius."[4] He was followed by Robert Harpur who had been trained at Glasgow, who held the chair of mathematics from 1761 to 1767, and who also taught physics. The wholesome combination of these related subjects was inherited from the English colleges and extended even beyond the middle of the nineteenth century. Thus in 1833 in a list of 58 college instructors in mathematics more than 50% were also recorded as giving courses in natural philosophy. This custom is also seen in the cases of Benjamin Peirce, Theodore Strong, and others. Not until 1859 was there a professor of pure mathematics at Columbia, William G. Peck holding the position, but changing to mathematics and astronomy in 1861. Although John Kemp seems to have been professor of

[4] The letter is now in the library of Columbia University.

mathematics from 1786 to 1799, from 1799 to 1812 he combined mathematics and natural philosophy. All of this work required a certain amount of equipment, but it is outside our present field to consider this matter in detail.

In astronomy such equipment seemed to be more essential; at any rate it attracted more attention. In 1760 Harvard was fortunate in receiving from James Bowdoin a planetarium, commonly known at that time as an orrery in honor of the Earl of Orrery, who was the first to order one made. Like various other universities (colleges) of the time, Harvard sent an expedition to New Foundland to observe the transit of Venus in 1769 which, as Jeremy Belknap[5] remarked, "was never before seen by mortal eyes but once." The College of the Colony of Rhode Island, generally called Rhode Island College (Brown University since 1804), made elaborate preparations for the observation of 1769 (the year of its first graduating class), having an unusually good set of instruments for that time.

### 7. LEARNED SOCIETIES AND SCIENTIFIC PERIODICALS

It was not in the colleges, however, that mathematics and the other sciences found their greatest encouragement in this period; it was in the learned societies and their publications.

The earliest noteworthy attempt to found a society that should embrace the ideals of the Royal Society of London was made by Benjamin Franklin, who was also the first American scientist to receive general recogni-

[5] 1744–1798. He published an *American Biography*, Boston 1794–1798, and (in 1792) a work on the discovery of America.

tion in England and France. When only twenty-one and still in the printing trade he organized in Philadelphia the "Leather-apron Club," which he always spoke of as "The Junto." With other members of this club, he was influential in founding (1743) the American Philosophical Society.[1]

President Gordon, of Magdalen College, Oxford, in his lectures on the literary relations of England and America, delivered at London University in 1931, remarked that "It was Benjamin Franklin who first opened English and European eyes to the New American man of which he was the first shining example." Although the speaker was referring to literature, the same may be said as regards science; aside from a few papers on astronomy, chiefly observational, America was little appreciated by European scholars before the later years of Franklin's life, or indeed before the latter part of the nineteenth century.

On August 15, 1765 a proposal was made to found an American Academy of Sciences.[2] The official minutes state that there were to be "Fellows of this Fraternity" who should be "resolved into five Orders of Literary[3] Merit—Philosophus—Legislator—Historius—Res gestae. Inventors. The three learned professions etc. Med & Theol. rank in the first; also Men of Eminence in Mathematic." It was recommended "That John Winthrop Esq$^{r}$ be the first president." The subject was

[1] E. P. Oberholzer, "Franklin's Philosophical Society," *Popular Science Monthly*, 1902, p. 430.

[2] These facts are taken from the original minutes, a photograph copy of which has been furnished us by Mr. F. E. Brasch of the Library of Congress, Washington. [redacted]

[3] "Public" is interlined above this word.

taken up again, as shown by the minutes, on November 24, 1766. In that year, and apparently as a result of this movement, there was established "The American Society Held at Philadelphia for Promoting and Propagating Useful Knowledge," but the venture was short-lived, the association being merged with the American Philosophical Society three years later (January 2, 1769). Among the members of the combined societies whose interests were in pure or applied mathematics were Benjamin Franklin; John Lukens, surveyor-general of Pennsylvania; John Sellers of Derby, Pa., a surveyor; John Winthrop (already mentioned), who succeeded Greenwood (1738) as Hollis professor at Harvard; Charles Mason (1730–1787), the London astronomer and surveyor who, with Jeremiah Dixon, ran the Mason-Dixon line of 39° 43′ 26.3″ N. between Pennsylvania and Maryland; Robert Patterson, a man of Irish birth, who came to America in 1762 and has already been mentioned; Benjamin Workman who, like Patterson, was on the teaching staff in the University of Pennsylvania; Jacques Alexandre César Charles (1746–1823) who did some important work in physics and aëronautics[4]; Walter Minto, already mentioned; and Cadwallader D. Colden (1688–1776), widely known as a scientist, first surveyor-general of New York, and lieutenant governor.[5]

As far as possible the American Philosophical Society was intended, as stated above, to do for the American colonies what the Royal Society was doing for the

[4] At the Conservatoire des Arts et Métiers.

[5] He wrote "An Introduction to the Doctrine of Fluxions," in *The Principles of Action of Matter*, London, 1751.

British Islands, a work which, in the nineteenth and twentieth centuries, it was destined to accomplish for the British Commonwealth of Nations. Its two publications, the *Transactions* (from 1771) and *Proceedings* (from 1838), were for a time the chief periodicals affording an outlet for mathematical achievements, still mainly in astronomy and physics.

Much more prominent than these societies was the American Academy of Arts and Sciences (1780), which has ever since been influential in maintaining a high standard of membership and of publications (the *Proceedings* and *Memoirs*). It was natural that such an organization, founded during the Revolutionary War should publicly state the intention "to give it the air of France rather than that of England."

The Connecticut Academy of Arts and Sciences was founded at New Haven in 1799 and has done much to advance the cause of mathematics. There were also numerous minor local societies, such as the Linonian Society at Yale (1753) which served a useful purpose, but their influence upon mathematics in the eighteenth century was negligible.

Having now considered some of the societies which aided the advancement of mathematics in the eighteenth century it is desirable to call attention to the publications of that European one, already mentioned, to which American mathematicians contributed most frequently during the colonial period. This was The Royal Society of London for Improving Natural Knowledge, generally known as The Royal Society. It is the oldest scientific society of Great Britain and one of the oldest in the world, and because of the racial, political, and

educational relations of most of the colonies to the Old Country, it was the one society to which mathematicians, physicists, and astronomers of the New World looked with the greatest respect. Founded in 1660, but having its remoter origin in a small group of "divers worthy persons" meeting in London as early as 1645, and in the Philosophical Society of Oxford (about 1648), it had for its first members men like Lord Brouncker and Sir Christopher Wren, both among the best known British mathematicians of the time, and the latter for his architectural ability in the rebuilding of London after the Great Fire. In its *Transactions* (from 1665) the first scientific memoirs of American scholars appeared, as will be shown later.

In spite of the fact that American mathematics showed little originality in the eighteenth century, it is interesting to see the encouragement given by the Royal Society to American scholars. In his memoir on the subject Mr. F. E. Brasch lists the names of seventeen Americans in the colonial period who were members of the Society, several of whose papers were thought worthy of publication. This brings us to certain more noteworthy stages in the development of American mathematics, which related, for reasons already stated, chiefly to articles on astronomy. Few of the latter, however, have any place in the history of mathematics, being chiefly concerned with ordinary observations of celestial phenomena and with the relatively simple computations involved. In the first five volumes of the *Transactions* of the American Philosophical Society (1771–1802)[6] the only mathematical articles worthy of

[6] Vol. I was reprinted in 1786

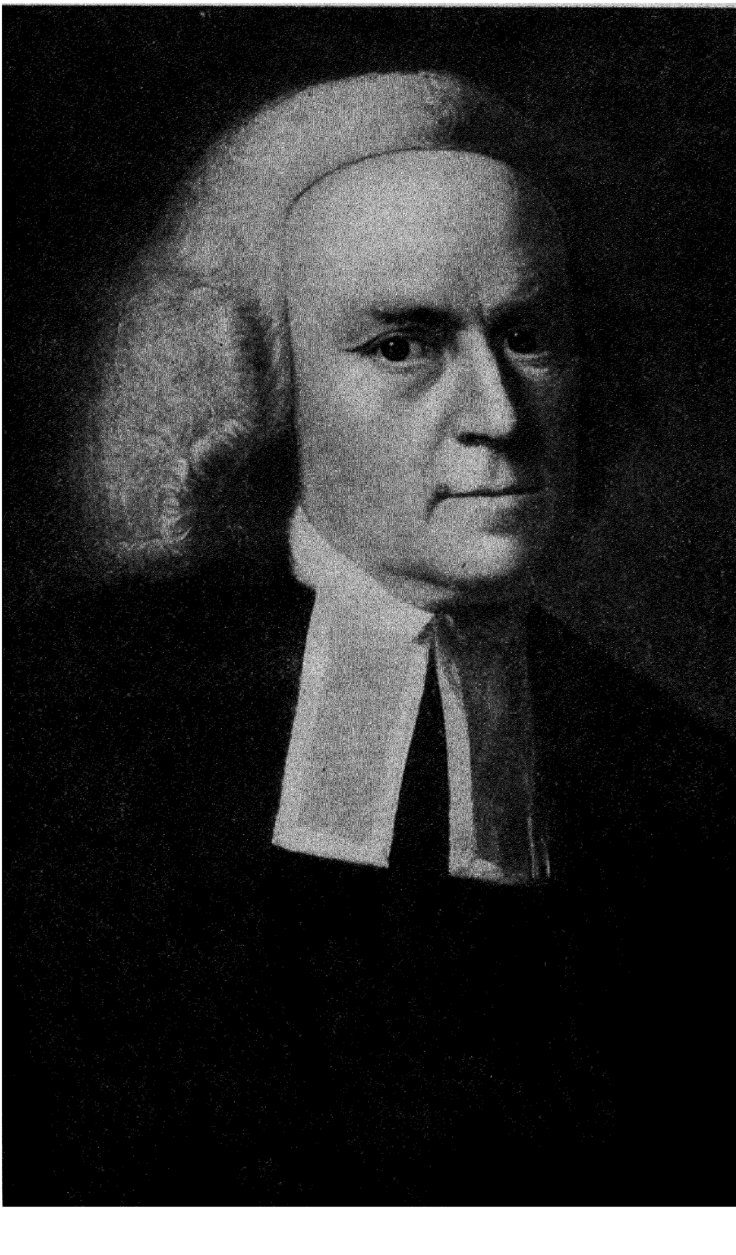
note were those of David Rittenhouse on (1) an optical problem of F. Hopkinson (II, 201), (2) a method of finding the sums of the several powers of sines (III, 155), (3) a "Method of raising the common logarithm of any number immediately" (IV, 69), and (4) converging series (IV, 21). Rittenhouse's better-known work will be considered later. As to the articles in the *Transactions* during this period, their value lay merely in the fact that they indicated an interest in certain phases of science, not that they showed any noteworthy originality.

There were, in England and America, various minor periodicals which contained lists of problems proposed for solution. Such opportunities have often been found exceedingly important in the development of mathematics. The most successful of these collections of problems was that which appeared a century later under the editorship of W. J. C. Miller in the *Educational Times*. Some of the leading mathematicians of the world were contributors to this notable section of a non-mathematical publication. In the eighteenth century there was nothing so elaborate as this, but such periodicals as *The Ladies' Diary* in England, edited at one time by Thomas Simpson, contained much good material, and it was this and others of its kind that suggested similar problem-solving columns in the shortlived *Royal American Magazine* (Boston, 1774), the *New York Magazine and Literary Repository* (1790–1795) and, in the early part of the century following, *The Enquirer or Literary, Mathematical and Philosophical Repository* (London, 1811–1813), to which Americans occasionally contributed.

## 8. PROMINENT NAMES

Although America produced no mathematicians of any marked prominence in the eighteenth century, there are two classes of scholars deserving of mention in connection with the development of the subject. These were, as in the preceding period, the astronomers and the several public leaders who promoted the study of mathematics.

Of the astronomers the two who seem to have appreciated the importance of mathematics the most were Winthrop and Rittenhouse. **John Winthrop** (1714–1779), the fourth of his name, a descendant of the first governor of the Massachusetts Bay Colony, was one of the most learned men of his time. He was only twenty-four when he was elected (1738) to the Hollis professorship at Harvard, and for more than forty years (1738–1779) he held this important position. One of the first things he did as a preparation for his duties was to master Newton's *Principia* and to make full use of the astronomical instruments which Hollis had sent from England. These included a telescope which had belonged to Edmund Halley (1656–1742) and which therefore represented a fairly late type. The original records of Winthrop's astronomical observations in 1739 are still in the Harvard Library. In 1740 he observed the first transit of Mercury ever seen in America and in 1761 he went to Newfoundland to observe the second one, this being probably the earliest purely scientific expedition sent out by any American colony. Evidence of his teaching of fluxions as early as 1756 has already been mentioned. His interests, however, were primarily in



John Winthrop (1714–1779)

the field of astronomy, and his memoir on the meteors of 1765 was communicated by Franklin to the Royal Society in that year. In the year following he was made a Fellow. Two years later (1768) he received the degree of LL.D. at Edinburgh and was made a member of the American Philosophical Society. He was the first to receive the same degree at Harvard (1773) and was the chief founder of the American Academy of Arts and Sciences.

Probably the best evidence that we have of the scholarship of Winthrop and, indeed, of the mathematical material available in America in his lifetime is to be found in his collection of books now in the library of Allegheny College. It includes more than a hundred volumes on pure and applied mathematics, practically all of them being well-known classics. For example, there are works by Barrow, Cassini, Cotes, De Lalande, Desaguliers, Descartes, Euclid, Gravesande, Halley, Huygens, Keill, Maclaurin, Maseres, Newton, Oughtred, Ramus, Whiston, and Wolf. Among these books are a considerable number which are now rare, and there is none which would not today be considered worthy of a place in any mathematical collection.[1]

In a letter written by Winthrop in 1764 the duties of a professor of mathematics and natural philosophy at that time are set forth as follows:[2]

My province in the College is to instruct the students in a system of Natural Philosophy and a course in Experimental, in

[1] We are indebted to Mr. F. E. Brasch, the biographer of Winthrop, for a copy of the catalogue of this library.

[2] N. Henry Block, "Certain ancient physical apparatus belonging to Harvard College," *Harvard Alumni Bulletin*, XXXV, 660 (1933).

which is to be comprehended Pneumatics, Hydrostatics, Mechanics, Statics, Optics, etc.; in the elements of Geometry, together with the doctrine of Proportion; the principles of Algebra, Conic Sections, Plane and Spherical Trigonometry, with general principles of Mensurations of Planes and Solids; in the principles of Astronomy and Geography, viz. the doctrine of the sphere, the use of the globes; the calculations of the motions and phenomena of the heavenly bodies according to the different hypotheses of Ptolemy, Tycho Brahe, and Copernicus, with the general principles of Dialling; the division of the world into various kingdoms, with the use of the maps, and sea charts; and the arts of Navigation and Surveying.

The second man in this period to make a reputation for himself in the field of astronomy, and in some degree that of applied mathematics, was **David Rittenhouse** (1732–1796).[3] From the trade of watchmaker he passed to that of maker of instruments of precision for scientific work in general. He also showed considerable ability in mathematics in his early years, taking up surveying as an avocation and assisting in establishing the Mason-Dixon line. The reduction of the data in this survey was the first astronomical computation of great importance in this country. He was appointed by the American Philosophical Society to observe the transit of Venus on June 3, 1769, and the degree of accuracy reached by him exceeded that of other American observers and equalled that of the best European astronomers. He also calculated to a high degree of accuracy the elements of the future transit of December 8, 1874, and the excellence of his papers in vols. I–IV of the *Transactions* of the Society led to his election as

[3] M. J. Babb, "The David Rittenhouse Bicentenary," *Sci. Monthly*, XXXV, 523–542. Portrait. See also R. C. Archibald's list of biographies in *Scripta Mathematica*, II, 83.

president (1790) in succession to Franklin. He was a Fellow of the American Society of Arts and Letters (1782) and of the Royal Society of London (1795) as a foreign member, and was given the degree of LL.D. at Princeton (1789). He was also vice-provost of the University of Pennsylvania and was known for his early use of the spider lines in telescopes.[4] In spite of all this, however, he was not a mathematician of any great ability. When the Marquis François-Jean de Chastellux (1734–1788) visited the university (1781) he commented on the fact that almost the only astronomical book in the library was an almanac. He also stated that Rittenhouse did not know enough mathematics to read D'Alembert, although possessed of unusual mechanical abilities. This judgment was probably fair, but his mathematical interests were sufficient to lead him to contribute to the *Transactions* of the American Philosophical Society the papers already mentioned. In 1792 Washington appointed him Director of the Mint, a position which he held until 1795, the year before his death. Thomas Jefferson's estimate of the achievements of Rittenhouse is given in the following statement relating to his construction of a planetarium:

> We have supposed Rittenhouse second to no astronomer living; that in genius he must be first because he is self taught. As an artist he has exhibited as great a proof of mechanical genius as the world has ever produced. He has not indeed made a world; but he has by imitation nearer approached its Maker than any man who has lived from the Creation to this day.

[4] This was not the first use, however, for Abbé Fontana had announced his invention of the device in 1775. See the *Astron. Jahrbuch der Ephemeriden*, Berlin, 1776, part 2, p. 100. See also the *Astron. Nachrichten*, XII (1835), 135.

These unwarranted superlatives are given rather to show Jefferson's interest in astronomy than to throw any light upon the work of Rittenhouse. About ten years later he wrote in more restrained language to his friend John Page, at one time governor of Virginia, referring to the latter's plan to place Rittenhouse on the faculty of the University of Virginia, and saying that he would be "an immense acquisition." Some years after the death of Rittenhouse he remarked in a letter to John Adams that he, "as an astronomer, would stand on a line with any of his time, and as a mechanician he certainly has not been excelled," thus showing the high esteem in which the foremost American of the time held one of the best known of his contemporary scientists.[5]

Of the great public leaders who did much for the encouragement of mathematics in the New World and the interchange of scientific knowledge between the scholars in Europe and America, two stand out as especially prominent. The first of these is **Benjamin Franklin** (1706–1790), a man whose genius and breadth of interests give him a permanent position in American history. Deprived of such education as the schools are supposed to give, at least beyond two years in the merest elements, he early resolved that greater facilities should be afforded for others. This led to the attempt in 1743 to establish an academy in Philadelphia, an effort which bore fruit in 1749 and eventually led to the founding of the University of Pennsylvania. Although he made no direct contributions to mathematics and never wrote a

[5] D. E. Smith, "Thomas Jefferson and Mathematics," *Scripta Mathematica*, I (1933), No. 1.

book on science, the encouragement which he gave to each was very great. His essay "On the usefulness of mathematics"[6] for example, although too elementary to demand our attention today, meant a great deal at the time it was written.

As an experimenter his reputation among European scientists began with his contributions to the study of electricity in 1746 and 1747 and particularly as to the nature of lightning in 1750, and his letters of 1751 to Peter Collinson had a profound effect. His spectacular arrival in France in 1776 on the *Reprisal* made him a kind of hero and increased his standing with European physicists. Fellow and medallist of the Royal Society, honorary doctor of Oxford, corresponding member of numerous learned societies, speaking four languages fluently and several others fairly well, his influence in the exchange of European and American thought was exceptional. This is seen in his presentation of John Winthrop's memoir to the Royal Society and in his attempt to induce Dr. Samuel Johnson to come to Philadelphia to direct English studies at the Academy of Pennsylvania. Some idea of monetary values at that time may be seen in the offer that he made to Johnson of £100 a year and traveling expenses. Coming in contact with the greatest scholars of Paris in such stirring times, he gave to America the opportunity of knowing the mathematicians of France whose works our scholars of that time were rarely capable of comprehending; and to France of knowing the feeble results of our own researches in the field of mathematics.

Speaking of the state of science and letters in Amer-

[6] *Pennsylvania Gazette*, Oct. 30, 1735.

ica at this period, the Abbé Guillaume Raynal[7] used these forceful but hardly well-considered words: "On doit être étonné que l'Amérique n'ait pas encore produit un bon poëte, un habile mathématicien, un homme de génie dans un seul art, ou seule science." This aroused Jefferson to defend his countrymen. He asserted that when the Americans shall have existed as a people as long as the Greeks had before they produced a Homer, the Romans before they produced a Vergil, the French before they had produced a Racine and a Voltaire, or the English a Shakespeare and a Milton, the question of why America had not produced enough great men might properly be considered. After speaking of Washington, as an example of an American of highest ability, he remarks that in physics the colonies had produced Franklin "than whom no one of the present age has made more important discoveries," an encomium quite as open to criticism as the stricture of the Abbé. The former quotation is of value as showing the opinion of the learned in Europe with respect to the New World, and the latter as showing the rather exaggerated opinion of a really learned American patriot with respect to the first scientist of much ability in this country. It is significant, however, that Jefferson selected a physicist rather than a mathematician to prove his case.

The second of the intellectual leaders who helped in the development of mathematics in the infancy of the science in America was **Thomas Jefferson** (1743–1826) himself, from whose letters we have already quoted. It

[7] 1713–1796, in his *Histoire philosophique et politique* (Maestricht, 1774, p. 92). See D. E. Smith on Jefferson, *loc. cit.*

is probable that no American of his time had a wider range of interest than he. His studies led him into the special fields of mathematics, astronomy, geodesy, and metrology as well as those of science in general, diplomacy, and statesmanship. Sent by Congress in 1784 to assist Franklin and John Adams in their negotiations with France, and succeeding the former as Minister Plenipotentiary, he remained until after the fall of the Bastille on July 14, 1789. These were the years in which young men of wealthy American families were occasionally sent to France to complete their education, and their influence together with that of Jefferson had doubtless much to do with the introduction of French mathematics, a generation later, into America. During his sojourn in Paris, Jefferson came in contact with Quesnay de Beaurepaire, who proposed a scheme of founding a European Academy in Richmond, Va., the professors being preferably French. The project aroused enough interest to secure subscriptions for more than $12,000. Although the approach of the French Revolution caused the plan to be given up, a similar one was projected by Sir Francis d'Ivernois (1757–1842). His idea was to transplant the Academy of Geneva to America, and the plan was seriously made in a memoir presented to John Adams, who had gone to the Court of St. James at the same time that Jefferson was appointed to the Court of France.

Jefferson's life in Paris gave him a lively interest in the new metric system and doubtless led to his attempt to simplify the complicated tables which we inherited from England. It seems also to have led him to a renewed interest in mathematics and its applications,

as witness his published articles on surveying, almanacs, astronomy, measures, the pedometer, and Napier's theorem respecting spherical triangles.[8] As to the last of these he wrote to Louis Hue Girardin in 1814 saying "I send you my formula and explanation of Lord Napier's theorem for the solution of right-angled spherical triangles. With you I think it strange that the French mathematicians have not used or noticed this method more than they have done. Montucla expresses a like surprise at this fact.[9] Yet he does not state the rule but refers it to Wolf, Cours de Math." He then proceeds to discuss the theorem and refers to such English works as those of John Potter,[10] Robert Simson (1687–1768), and Charles Hutton (1737–1823).[11]

Jefferson claimed that the rule was not, as Hutton had asserted, too difficult for beginners, and he showed that it had the advantage of being usable for oblique as well as right-angled triangles, referring also to the work of Lord Buchan and Dr. Minto on the life of Napier. In the same letter he speaks of his method of plotting the courses of a survey, which "is so obvious and simple that as it occurred to myself, so I presume it has to others although I have not seen it stated in any of the books." In spite of this manifest interest in the subject he speaks of his contributions as "mathematical trifles" that were for him only an amusement.

[8] These papers appear in the Monticello edition of his *Works* (20 vols., Washington, 1904, 1905), vols. III, 26, 32, 50; IV, 42; V, 36, 91, 157, 303; VI, 460; VIII, 48; XII, 313; XIII, 95; XIV, 21, 121, 128, 484; XV, 148, etc.

[9] *Histoire des Mathématiques*, 2d ed. II, 25.

[10] *A System of Practical Mathematics*, London, 1753.

[11] His *Course of Mathematics* (London, 1798–1801) went through at least five American editions.

Although it may seem that Jefferson was not a mathematician in the sense that we shall use the term, and that therefore there may be some objection to quoting at greater length from his letters, there is a valid reason for adding to what has been said concerning his encouragement of the subject under consideration. We therefore include a few sentences from a letter written by him in 1799 and only recently published.[12]

Dear Sir Monticello June 18. 99..

I have to acknolige the reciept of your favor of May 14. in which you mention that you have finished the 6. first books of Euclid, plane trigonometry, surveying & algebra and ask whether I think a further pursuit of that branch of science would be useful to you. there are some propositions in the latter books of Euclid, & some of Archimedes, which are useful, & I have no doubt you have been made acquainted with them. trigonometry, so far as this, is most valuable to every man. there is scarcely a day in which he will not resort to it for some of the purposes of common life; the science of calculation also is indispensible as far as the extraction of the square & cube roots, Algebra as far as the quadratic equation & the use of logarithms are often of value in ordinary cases: but all beyond these is but a luxury; a delicious luxury indeed; but not to be indulged in by one who is to have a profession to follow for his subsistence. in this light I view the conic sections, curves of the higher orders. perhaps even spherical trigonometry, Algebraical operations beyond the 2[d] dimension, and fluxions. there are other branches of science however worth the attention of every man. astronomy, botany, chemists natural philosophy, natural history, anatomy. not indeed to be a proficient in them; but to posess their general principles & outlines, so as that ve may be able to amuse and inform ourselves further in any of them as ve proceed through life & have occaseon for

[12] *Scripta Mathematica*, vol. I, No. 1, 1933. The letter, about one third of which is here published, was called to our attention by Professor Witmer, Librarian of Teachers College, Columbia University, having been for many years in the library of the College, practically unknown.

them. some knowlege of them is necessary for our character as well as comfort. the general elements of astronomy & of naturil philosophy are best acquired at an academy where ve can have the benefit of the instruments & apparatus usually provided there: but the others may vell be acquired from books alone as far as our purposes require. I have indulged myself in these observations to you, because the evidence cannot be unuseful to you of a person vho has often had occasion to consider vhich of his acquisitions in science have been really useful to him in life, and vhich of them have been merely a matter of luxury.

This letter in full constitutes one of the earliest efforts, made by a statesman of distinction, to express his views on the study of mathematics in America.

## 9. SUMMARY OF CONDITIONS IN THE EIGHTEENTH CENTURY

The judgment passed by President Barnard upon the scientific achievements in this country at the close of the eighteenth century has already been mentioned. To this it is desirable to add the opinions of certain foreign writers[1] upon the work of the Philosophical Society. A writer in the *Edinburgh Review*, speaking of Volume V of the *Transactions*, had this to say of the contents:

This volume . . . will not, we apprehend, repay the labour of him who may be induced to wade through it. . . . Of all the academical trifles which have ever been given to the world, these pages are the most trivial and dull. . . . We have dwelt longer upon this article than its merits justify . . . for the purpose of stating and exemplifying a most unaccountable fact—the scarcity of all but mercantile and agricultural talents in the New World..

[1] For a discussion of this general topic of criticism see W. B. Cairns, "British Criticism of American Writings, 1783–1815," *Univ. of Wisconsin Studies in Language and Literature*, No. I. Madison, 1918. The two following quotations are from this study. See also No. 14, 1922.

Somewhat less dignified, but expressive of the British sentiment then and later is this note from the *Anti-Jacobin Review:*

> Contemptible in point of talent, as we have always understood the *soi-disant* 'Philosophical Society' to be, we are not astonished to find the greatest (and, by far, the best) part of its transactions consisting of articles contributed by British writers. . . . As to the native productions, they are, both in matter and in manner, altogether beneath the attention of criticism.[2]

By way of rebuttal an equally undignified utterance by De Witt Clinton may be found in the *Transactions of the Literary and Philosophical Society of New York*,[2] where the *London Quarterly Review* is charged with "an impotent effusion of malignity against our country, its morals, manners, intellect and institutions" which are asserted to have been "invented and propagated by ignorant and insignificant tourists."

From these statements we may fairly conclude that modern mathematics was substantially unknown in America in the eighteenth century, but that questionable manners then and later existed both in the Mother Country and in her offspring.

[2] Vol. I, p. 79, New York, 1815.

# CHAPTER III

## THE NINETEENTH CENTURY. GENERAL SURVEY

### 1. THE COLLEGES AND UNIVERSITIES

The first three centuries of our history were, as we have seen, barren of achievement in the domain of mathematics. The first half of the nineteenth century was a time of preparation for action. The third quarter closed with evidence that a period of awakening was at hand. The last quarter saw the beginning of the period which, in the first third of the twentieth century, placed the mathematics of the New World in a more favorable position than had been anticipated either in Europe or in America. With this latest development the present work is not directly concerned.

We now propose, in this chapter, to take a general survey of the work of the nineteenth century, as we have of the two preceding ones, and to devote Chapter IV to some of the branches of mathematics to which scholars in this country, particularly in the period between 1875 and 1900, gave special attention.

The university, in the present sense of the word, is a relatively modern institution in America, and even yet the term is loosely used. Many of our American universities were colleges in fact long after they assumed the more pretentious name, and some of them are such at the present time. Therefore the university name does not indicate that students did or could carry on graduate work in mathematics in any particular institution

which bore the title. So far as this branch of learning is concerned, it was not until after Johns Hopkins opened its doors in 1876 that university instruction in advanced mathematics was systematically and successfully offered to any considerable number of students. We shall therefore begin by mentioning the situation of mathematics in the type of educational institution inherited from the period of the American Revolution.

During the first half of the nineteenth century there was a rapid development of small colleges in this country, each representing as a rule some religious sect and having for its purpose the training of clergymen or the education of boys in the tenets of the particular church. In spite of this limited vision of the meaning of a liberal education, these colleges gave at least some knowledge of the nature of elementary mathematics. In 1827 George Bancroft[1] wrote to an English friend concerning the condition of college education in this country. Since he had studied in German universities his knowledge of European standards was unusually good. He remarks:

> The progress of education in our country has in the last few years surpassed all expectations. . . . In our schools for higher instruction a great reform has taken place. . . . At present very respectable Colleges are rising up in the interior, even beyond the Alleghany Mountains. And in the mountain district there is almost a line of colleges, extending through the Union: in Georgia at Athens, in S. Carolina, at Columbia, in N. Carolina at Chapel Hill, in Virginia at Charlottesville; in Pennsylvania there are several; in New York at Hamilton, & Geneva.[2]

[1] 1800–1891. United States Minister at London (1846–1849) and later (1867–1874) at Berlin.

[2] The letter is now in the D. E. Smith Collection, Columbia University Library.

This rather optimistic statement gives first-hand testimony of the rapid rise of colleges, but the question which concerns the development of mathematics relates rather to the courses of study open to the students than to the multiplicity of such generally mediocre institutions. The results of the study of these courses are discouraging. Few carried the subject beyond the limits set by the better secondary schools. In spite of the college name which they assumed, President Barnard of Columbia stated in his *Analysis of Some Statistics of College Education* (New York, 1870) that he found only 220 institutions doing any kind of college work, with only 17,500 students, or about 80 on an average to each one, or 20 to each year. In general these small colleges were sectarian, as the larger ones had been in their early days. In the University of Georgia, for example, from 1801 to 1885 all but one of the presidents or chancellors were doctors of Divinity. The training offered was distinctly that for the preacher, and it was usually by a preacher or by some politician out of office that mathematics was taught. Such a state of affairs was not confined to the small and feeble colleges. For example, in the University of Tennessee (founded in 1793), the "Honorable Horace Maynard" was (1840) announced as tutor and principal in the preparatory department; the next year as teacher of mathematics and ancient languages; the next year as teacher of ancient and modern languages, and in 1843 as "professor of mathematics, rhetoric, and belles lettres." Happily it is recorded that he resigned in 1843.

Even if the professors had been adequately prepared for the work of stimulating the students in their charge,

which they were not, the programs assigned to them in the smaller colleges gave them no time for any serious study. For example, Norwich University (now at Northfield, Vt., but formerly a military school) had on its faculty in 1836 a man well known in his day, Zerah Colburn, who was professor of mathematics, English literature, Latin, Greek, French, and Spanish, while in the same year the vice-president was professor of mathematics, natural philosophy, civil engineering, topographical drawing, and military instruction.[3] Even our worst grade of high school could hardly show a more lamentable situation.

The reason why such a state of affairs could have existed only a century ago is due in part to the fact that the professors in the sectarian colleges were, in a sense, missionaries, and with the zeal of propagandists they would undertake to teach the tenets of their faith for a pittance and take on duties for which they were not at all prepared. The standard of salary was therefore low and it influenced college salaries in the better institutions. Even at Bowdoin, in the middle of the century, Professor Bushnell's salary for teaching mathematics was "$600, if we can raise it," and in general a salary of $1500 in the second quarter of the century was thought to be generous, although of course the difference in the purchasing power of the dollar between 1850 and the present time must be recognized.

Lest it be thought that these illustrations do not represent the situation fairly, it should be repeated that a professorship of mathematics *per se* is a modern office

[3] See W. A. Ellis, in his history of the university, 1911.

in America. There was not enough mathematics known in America a century ago to seem to warrant such a position. As already stated it was not unusual for a man to be a professor of mathematics combined with astronomy, or physics (natural philosophy), or even medicine or theology. Thus at Yale, from 1800 to 1836, the five professors of mathematics (Meigs, Day, Fisher, Dutton, and Olmsted) were also professors of natural philosophy, and the records of ΦBK show a similar situation for the thirteen professors of mathematics listed among the members enrolled from 1812 to 1840. At Columbia University Robert Adrain combined mathematics and physics (1813–1820) and later replaced the physics by astronomy (1820–1825). His successor, Henry James Anderson, went still farther and combined (1825–1843) mathematics, physical astronomy, and mechanics. Many other similar cases might be cited, and the notable one of Benjamin Peirce will be mentioned later.

There was, of course, much to be said in favor of combining mathematics with physics and astronomy, as was so frequently seen in Europe. In some cases such a breadth of view produced scholars of high standing. This is seen in the case of F. A. P. Barnard who was professor of mathematics, natural philosophy, and astronomy in Alabama University (1837–1848) and later of chemistry and natural philosophy (1848–1854), then proceeding to the University of Mississippi as professor of mathematics and natural philosophy (1854–1856), becoming chancellor in 1856, and going north at the outbreak of the civil war. He was connected with the United States Coast Survey for a time, after which he

became president of Columbia College, where he laid the foundations for the University.

As to mathematical requirements for entering college, they were practically non-existent at the opening of the century. Harvard's first step in this direction was made in 1803 when she required the mere rudiments of arithmetic as far as the Rule of Three. In 1816 the whole of elementary arithmetic was required, and in 1819 a slight knowledge of algebra was added, and this was not changed materially for fifty years.[4] Not until 1837 was arithmetic dropped from the freshman course. The curriculum was rigidly fixed and if a man in 1824 wished to study fluxions and chemistry he could do so only by substituting these for Syriac and Hebrew. The result of the raising of such obstacles accounts in part for the fact that out of two hundred mathematical theses written by the juniors and seniors at Harvard in the period 1782–1815, only one was written on fluxions before 1803. During the next thirteen years, however, this number increased to eighteen, but even this was only 9% of the total for the thirty-four years.

The situation in other colleges of high rank was substantially the same. For example, as late as the middle of the century Dartmouth required for entrance only arithmetic and Bourdon's algebra through linear equations.

As to the curriculum in mathematics, the following subjects were taught at Dartmouth and Yale (selected as fair types) at the end of each of the first two quarters

[4] G. G. Bush, *History of Higher Education in Massachusetts*, Washington, 1891, p. 151.

of the century, approximate as to dates and varying slightly as to topics:

1825. Freshman—arithmetic and algebra; Sophomore—Euclid, plane trigonometry, mensuration, surveying, navigation; Junior—conic sections, spherics, natural philosophy, spherical trigonometry, astronomy.

1850. Freshman—plane geometry, algebra finished; Sophomore—trigonometry, surveying, mensuration, analytics, calculus; Junior—natural philosophy, astronomy.

It will thus be seen that about all the colleges attempted to do for the student who wished to continue beyond the merest elements was to substitute some mathematical subject for a language like Hebrew in the senior year. In Columbia Professor Anderson, a physician, gave lectures (1835) to the senior class on the integral calculus. Part of his notes on the subject are now in the New York Public Library, and they represent about what any high-school teacher might give to his senior class today if allowed to do so.

It is evident from these specimen courses of study that astronomy was still considered the culmination of the work in mathematics in the first half of the century. Further evidence of this situation is seen in the fact that of all the theses on mathematics required in the last two years of the Harvard course in the period 1782–1836 approximately 68% were on astronomy, while only 9% were on algebra and 8% on geometry or closely related topics.

It will therefore be observed that the work in the freshman year of college was, in the first half of the century, merely that of a mediocre high school of the

twentieth century, a fact which testifies to the poor work done in the field of mathematics in the preparatory schools of the time. Even in the fourth decade of the nineteenth century the situation, relatively speaking, was about the same; that is, a considerable part of the work done in the freshman and sophomore years of college could have been quite easily done in the same length of time in the high schools, and in due time it will be so done in special mathematical classes. In algebra, geometry, and trigonometry, as in most other school studies, the curriculum is still burdened with traditional material of little value to pupils of ability in the subject, and the same may be said as regards the college work in the calculus and analytic geometry. The late Professor J. C. Fields, (1863–1932) of the University of Toronto, in an address published just before his death, spoke very truly when he remarked, "One would not be far wrong in attributing the near sterility of mathematicians on this continent in the last generation to the teaching of the calculus." The common custom of fifty years ago of "burning the calculus" at the close of the sophomore year was a perfectly justifiable display of the students' estimate of the value of the subject as then taught.

As to the facilities for mathematical study at the close of the first quarter of the century, and the quality and ambitions of the students, the records of Middlebury College, Vermont, which was chartered in 1800 and represents fairly well the average college in Barnard's list of 220, give us the following information:

1. As to the quality of students: "If any scholar shall assault, wound, or strike the President, or a Tutor, or shall maliciously

or designedly break their windows or doors, he shall be expelled."

2. As to facilities for study, there were eighteen mathematical books in the library in 1823. To take a folio book from the library cost the student 7¢; a quarto, 6¢; and an octavo, 4¢.

3. As to any work in mathematics beyond the algebra requirement, this could be done only by private instruction.

Of course the older and better equipped colleges must not be judged by this statement, and yet their library facilities seem to us to have been pitifully small. As regards mathematics this was particularly the case, for the greater part of each early college library was still merely theological and classical. A list of volumes in the libraries "of the principal Colleges and Academies of the United States" was published at Middletown, Conn., in 1827, the numbers in the largest college libraries being as follows:

Harvard 28,000 (by far the largest); Yale, 9000; Princeton, 8000; Dartmouth, 6000; Bowdoin, 6000; Brown, 5000; Union, 5000; South Carolina, 5000; Washington 5000; Columbia, 4000. Today Harvard is approaching the 3,000,000 class; Yale, the 2,000,000; and Columbia the 1,500,000, two of these being supplemented by large municipal libraries and by private collections available for use by advanced students; for example, those of Boston, Chicago, and New York raise the total number of books that can be consulted by a student in any one of these cities to several million.

The mathematical library equipment of each of our large universities is today better than in most of the universities of Europe.

As to the mathematical books published in America at the opening of the century there is some slight information in *A Catalogue of all the Books printed in the United States* (Boston, 1804). Twenty-nine items appear

of which six are on astronomy, surveying, or navigation, the rest being on arithmetic or its simple applications. Of the six mentioned, not one showed any scholarship beyond the mere rudiments.

Lest it may seem from the above list of libraries and from the work done at Harvard and Yale that it was only in the northern colleges that mathematics prospered in the early part of the nineteenth century, it should be made clear that this was by no means the case. The influence of Jefferson gave to the University of Virginia the high rank in the South that Harvard had in the North, and its offering in mathematics was for a time unsurpassed anywhere in America. In 1825 Thomas Hewitt Key was called from Cambridge, England, to be professor of mathematics and to bring the traditions of Trinity College to the New World. After two years he was succeeded by Charles Bonnycastle (*c.* 1797–1840). The course of study prepared by the latter was probably the best in the country. Even if only a few were able to complete it, the mere fact that someone envisaged its possibility was in itself a good omen. The students were expected to use some of the best English and French textbooks, to study the mechanics of Giuseppe Venturoli (Bologna, 1806, or later editions), and the *Mécanique Céleste* (Paris, 1799–1825) of Laplace. The course included the theory of projection and the study of curves and surfaces, while the calculus was enriched by what was then the best modern material from English sources. Bonnycastle's own contributions to the literature of the subject were, however, rather trivial, aside from a critical examination of Taylor's theorem.

Bonnycastle was succeeded, after a short interval, by J. J. Sylvester (1841) who, although a very promising young mathematician, failed as a teacher and left at the end of a year. Thirty-four years later he returned to America and made Johns Hopkins for a time the mathematical Mecca of the New World. A few years after Sylvester left Virginia Charles S. Venable (1827–1901) was called to take up the work, and with some of his colleagues made a number of contributions of an expository nature. With him on the faculty was the astronomer Ormond Stone (1847–1933), founder and one of the editors of the *Annals of Mathematics.*

The situation in Canada, so far as pure mathematics was concerned, did not differ materially from that in the United States. In Professor J. C. Fields' address to which reference has already been made, he gave as the causes of the poor work in both countries (1) The defective mathematical training in the secondary schools and (2) The defective textbooks, particularly in the calculus. He called attention to the fact that in Canada the work in physics has been better than that in mathematics, due to the influence of England, whereas in the United States the influence of Germany has led to the placing of the emphasis on pure mathematics.[5]

## 2. EUROPEAN INFLUENCES

During the War of Independence it was natural that there should be a distinct lessening of British influence upon the schools of those colonies which were to become the United States. France had interests which led her to take the part of the non-British colonies, and

[5] *Anniversary Volume, Roy. Soc. Canada, 1832–1932*, pp. 107–112.

there were strong ties which bound her to the oldest part of Canada. New England and her southern neighbors could not conveniently send their young men to England during the war, and most of their people did not wish to do so after peace was secured. Louisiana was under French influence for a considerable time and the mission schools were generally established by French priests. It was therefore natural that French influence on mathematics should tend to increase and British influence to decrease in a large part of settled America.

Even before the Revolution the American Philosophical Society was more closely related to France than to England, having among its members Buffon (1768), Le Rey (Le Roi, Le Roy; 1773), Lavoisier (1775), and Condorcet (1775), whereas in the same period only Maskelyne (1771) was elected from England. During and after the Revolution there were added to its rolls Luzerne (1780), Lafayette (1781), Barbé Marbois (1781), Jacques Alexandre César Charles (1786), Cabanis (1786), and a considerable number of others, chiefly those who had assisted the colonies in their struggle.[1] In 1783 the Society, on the motion of Jefferson, directed Rittenhouse to make an orrery to be presented to the King of France and the records show numerous other gestures tending to strengthen French influence in the new republic. Franklin, who was himself so sympathetic with France, left as a legacy to the Society ninety-one volumes of the history of the Académie des Sciences.

[1] *Early Proceedings of the American Philosophical Society*, Philadelphia, 1884.

Among the ways in which the French influence was powerfully felt in this country, may be mentioned the translations of textbooks on elementary geometry, elementary algebra, mathematical astronomy, and descriptive geometry.[2] As to elementary geometry the French work which influenced the schools of the United States the most was the *Éléments de Géométrie* (1794 and later editions) of Adrien Marie Legendre (1752–1833). It was first translated (1819) into English by John Farrar (1779–1853), who was Hollis professor of mathematics at Harvard from 1807 to 1836, and who also included in it Legendre's *Traité de Trigonométrie.* Three years after its publication a translation of the geometry was made anonymously by Thomas Carlyle and edited by Sir David Brewster.[3] This was revised by an American publisher, James Ryan,[4] to meet the needs of the West Point Academy. Its greatest success was due, however, to a revision by Charles Davies.[5] It was later revised by J. H. Van Amringe, sometime Dean of Columbia College, this revision passing through about thirty editions. Not until George A. Wentworth's *Elements of Plane Geometry* appeared in 1877 was there any very successful rival to Legendre, and this one succeeded because of its page arrangement rather than because of any important change in the sequence or the selection of propositions.

[2] On this and other related matters see Lao G. Simons, "The Influence of French Mathematicians at the end of the Eighteenth Century upon the Teaching of Mathematics in American Colleges," *Isis*, XV, 104.

[3] Edinburgh, 1822.

[4] New York, 1828. It was he who started the shortlived *New York Mathematical Diary* in 1832.

[5] 1798–1876. He was a West Point graduate of 1815 and was professor there from 1816 to 1837 and at Columbia from 1857 to 1865.

The elementary algebra which did for this subject what Legendre's did for geometry was the *Éléments d'algèbre* of Pierre Louis Marie Bourdon.[6] This was translated by John Farrar in 1831 and later by Edward C. Ross, a graduate of West Point and for a time an assistant professor there, and professor of mathematics and natural philosophy at the New York Free Academy which had been founded in 1847 and which became the College of the City of New York in 1866. The book had already been translated in part, along with the *Éléments d'algèbre* (1799) of Sylvestre François Lacroix (1765–1843), and embodied in William Smyth's *Elements of Algebra.*[7] The third translation (aside from Smyth's adaptation of it) was made by Charles Davies, this version having a very large sale.

The algebra of Lacroix, mentioned above was translated in 1818, and therefore before Bourdon's work. This was probably done by Farrar, and it went through five editions. Dr. Simons states that it was "the first work translated by an American for use in American colleges." Farrar also translated or adapted Lacroix's trigonometry and his arithmetic, and the calculus (1824) of Étienne Bézout (1730–1783), and in general he based all his works on French models.

Davies also made much use of Bourdon's *Application de l'algèbre à la géométrie*[8] in his *Analytical Geometry* (1836), a work which was several times republished and which had considerable influence in American colleges.

The following tribute by Benjamin Peirce was paid

[6] 1779–1854. Paris, 1817 and later editions.

[7] 1830, and five other editions.

[8] Paris, 1824; 2d ed., 1828.

to the various translations and adaptations of the French textbook writers, the more interesting because Peirce himself had written an algebra in 1837 which had not proved very usable in the schools:

"The excellent treatises on Algebra . . . containing as they do the best improvements of Bourdon and the other French writers, would seem to leave nothing to be desired in this department of mathematics. . . . The investigation of each proposition has been conducted according to the French system of analysis."

Little evidence is needed to show that the French works of the early part of the nineteenth century furnished more of modern mathematics than did those produced in England. Harvard recognized this, as we have seen, in the translations made by Farrar; but, as we have also seen, the military schools at West Point (opened in 1802) and in Virginia recognized it even more. Unlike all other American colleges of the first half of the century, West Point specialized in mathematics, due in part to the general influence of Jefferson, but particularly to the special efforts of Lieutenant-Colonel S. Thayer, who was superintendent from 1817 to 1833. Charles Davies presented one of his books to Thayer with this inscription: "In the organization of the military Academy under your immediate superintendence, the French methods of instruction, in the exact sciences, were adopted; and near twenty years experience has suggested few alterations in the original plan."

It is due to this that the *Essai de Géométrie analytique* (1805) of Jean Baptiste Biot (1774–1862) was used for a number of years, and in its French form, at West

Point, beginning as early as 1821. It was translated into English in 1840 by Francis H. Smith, at one time an assistant professor there, and later a professor at the Virginia Military Institute. Smith also translated (1868) the *Éléments de trigonométrie* (1830 and later) of Lefebure (*c.* 1785–1869). In 1842 he wrote a work on mathematics in which the algebra portion was taken largely from the *Cours d'analyse algébrique* (2d ed., 1803) of Jean Guillaume Garnier (1766–1840), and that on curves of the fourth degree from the latter's *Traité du calcul différentiel et du calcul intégral* (1797 and later).

Not only were French textbooks influential in America, but occasionally French scholars came to this country to make their work known in our schools. For example, Claude Crozet, a graduate of the École Polytechnique, a school which inspired much of the work at West Point, was for a time professor of mathematics at the latter school and published a *Treatise on Descriptive Geometry* (1821). Charles Davies also wrote a work (1826) on the same subject and this went through several editions.

In the field of mathematical astronomy the outstanding work of the first half of the century was the translation made by Nathaniel Bowditch of Laplace's *Traité de Mécanique Céleste*, which will be mentioned later. Mention should also be made of the strong bonds of sympathy between France and the southern states, leading the educated classes in that region to favor the French rather than the English texts. An article on this subject in the *Southern Review*[9] affords some inter-

[9] Charleston, 1828, I, 107–134.

esting first-hand information upon the situation. M. F. Baldensperger has discussed the matter from a different angle. He asserts that the considerable body of refugees who came to this country after the French Revolution and the First Empire, formed national groups in cities like Boston, New York, and Philadelphia, mingling but slightly with Americans;[10] this, however had little bearing upon the influence of French science in America.

Not only was the nineteenth century indebted to France but it also was under obligations to other continental countries for the contributions made by several scholars who found at least a temporary home in the new world. This is especially true in the field of mathematical physics and astronomy. Among these scholars was Father Angelo Secchi (1818–1878), a Jesuit, who was exiled from Italy in 1848 and spent two years in the Georgetown University, where he became professor of physics and mathematics and director of the observatory. He returned to Rome in 1850. He was a voluminous writer, among his American contributions to mathematics being one in 1849 on the curve described by a movable pulley.[11]

Father Benedict Sestini (1816–1890), also a Jesuit and like Father Secchi exiled from Italy in 1848, was for twenty years a member of the faculty of Georgetown University. Although primarily an astronomer, he wrote a number of textbooks on mathematics. A third member of the Jesuit order who contributed no-

[10] See his *Le mouvement des idées dans l'émigration française, 1789–1815* (Paris, 2 vols., 1925), and *Lés réfugiés bonapartistes en Amérique, 1815–1830* (Paris, 1923). On the influence of France in our naval courses, see Park Benjamin, *The United States Naval Academy*, New York, 1900.

[11] *Silliman's Journal*, vol. VIII.

tably to the bibliography of higher mathematics of his time was Father J. G. Hagen whose work is mentioned later.

Although we were greatly influenced by the French treatment of algebra, geometry, trigonometry, and astronomy, our arithmetic remained essentially British in its presentation, and our calculus was quite generally Newtonian in name (fluxions) and in content until after the half-century line was reached. Our early work in conics was also of the Greek type as modified by the eighteenth-century British writers. The historian and diplomat, George Bancroft, referred to textbooks in his letter of 1827 already mentioned, in these words: "I can say this with a little pride: we are very attentive in America to all that is done in England for the general good." The impression conveyed is correct as to certain kinds of books—the classics, textbooks for the elementary schools, fluxions, and conics—but not, as has been shown, for the more modern mathematics.

Of the large number of textbooks in elementary mathematics published in this country, few have done much to give real insight into the subject or to show the best of the foreign influence. Probably the first to advance materially the study of elementary geometry and to reflect this influence in a notable way was William Chauvenet's (1820–1870) *A Treatise on Elementary Geometry* (Philadelphia, 1870) with a later edition by W. E. Byerly of Harvard (1849—). Not only was his treatment of Euclidean geometry considerably above the average, but the work contained an introduction to modern geometry. Although not well written from the standpoint of teaching, it was recognized by mathe-

maticians as a marked advance on the current books of its time. His *Treatise on Plane and Spherical Trigonometry* (Philadelphia, 1850) was also well received, being supplemented by articles in Gould's *Astronomical Journal* (1851–1854).[12]

A similar influence was exerted by William Mitchell Gillespie (1816–1868), a graduate of Columbia in 1834, who became a professor of civil engineering in Union College eleven years later. Under the title of *The Philosophy of Mathematics* he published in 1851 a translation of part of the *Cours de Philosophie Positive* of Auguste Comte. He also published a pamphlet setting forth the nature of the work required for entering the École Polytechnique at Paris, and in two works on road making and surveying he made use of French methods.

Occasionally unexpected instances of the influence of European mathematics are found in more humble quarters. For example, Carl Wasmund, who is described as a surveyor, a native of Stralsund, in Pomerania, but living in Black Earth, Wisconsin, in 1859, contributed three articles to *Grunert's Archiv*, two involving the theory of combinations and combinatorial products.[13]

## 4. SCIENTIFIC SOCIETIES AND PERIODICALS

The learned societies established in the United States and Canada in the nineteenth century were little concerned in the first half of this period with pure mathe-

[12] His *Manual of Spherical and Practical Astronomy* (2 vols., Philadelphia, 1863, with later editions) was highly esteemed in Europe as well as in this country.

[13] See XIII, 96; XXI, 228; XXIV, 440. Also see *Bibl. Math.*, I (1873), 130.

matics, although those devoted to philosophy and science offered an opportunity for presenting such rather elementary mathematical papers as were offered by their members. Some of these societies were local, like the United States Military Philosophical Society, established at Washington in 1808, of which Jefferson was a member. There was also the Literary and Philosophical Society of New York, established in 1814 under the presidency of De Witt Clinton, then the mayor of the city. This served as the chief means for the interchange of local information regarding mathematics and the physical sciences, and numbered among its "counsellors" Robert Adrain, Robert Fulton, and Cadwallader D. Colden, and among the members Andrew Ellicott, professor of mathematics at West Point. Ellicott's influence was naturally considerable, owing to the standing of the mathematical department at West Point, as mentioned on page 79. The attitude of DeWitt Clinton with respect to foreign critics of the Philosophical Society may be inferred from his contemporary remarks, as already stated (page 64).

Besides the societies mentioned in Chapter II, others of a general nature, offering facilities for the reading of mathematical papers, were formed in the nineteenth century. Prominent among these was The American Association for the Advancement of Science,[1] in which Section A is concerned with mathematics and Section D with astronomy. At the annual meeting of this society the vice-presidents, who are the presiding officers of the various sections, present papers on some phase of the recent development of their subjects, or on some re-

[1] Founded in 1848, incorporated in 1874.

lated topic. The society is modeled after the British Association for the Advancement of Science as the Royal Society of Canada is on that of London. The Royal Society was founded in 1831 having for its object the fostering of the spirit of research.[2]

As to the publications of these associations, the American Academy of Arts and Sciences began the issue of its *Proceedings* in 1846; The National Academy of Science, its *Memoirs* in 1866; the Royal Society of Canada, its *Proceedings* and *Transactions* in 1882, and the American Association for the Advancement of Science publishes *Science* and the *Scientific Monthly*. Each of these publications often contained papers on mathematical topics. Lists of these periodicals are given in the *Union List of Serials in Libraries of the United States and Canada* (New York, 1927).

The early periodicals devoted in whole or in part to mathematics were, as a rule, so puerile as to be hardly worth mentioning. The *Saturday Evening Post*, founded by Franklin in 1728, was an exception to this rule, since its problem column, beginning in 1806, ran for fifty years, although with numerous breaks in the series. In 1862 the plan was adopted of publishing problems weekly, Artemas Martin being the leading contributor.

About the beginning of the 19th century several mathematical or semi-mathematical journals appeared, most of them very short-lived. The first of these journals was *The Mathematical Correspondent*, begun as a quarterly in 1804. In the preface the editor, George Baron, expressed his view as to the status of American

[2] See the address by the late J. C. Fields, published by the University Press, Toronto, 1932.

mathematics in these words: "When we consider the great exertions of learned men to disseminate mathematical information in other countries we must be surprised to find that this kind of knowledge is most shamefully neglected in the United States of America." A correspondent remarks (Vol. I, p. 93):

> In this country authors of arithmetic have lately sprung up like a parcel of mushrooms, and it would have been well for the rising generation had the former been so harmless as the latter. These upstart authors have most perniciously corrupted, distorted and degraded the noble and useful science of numbers and metamorphosed our sons into counting machines, moving according to a heterogeneous collection of unscientific and stupid rules. A good book of arithmetic is wanted in this country; but so long as the wretched productions of Pike Walsh Shepherd and Co are encouraged, we cannot expect a man of talents to enroll his name in our list of numerical authors.

The tone of the *Correspondent* under the guidance of Baron, a man who showed his ignorance continually, was very low. The editor and certain contributors indulged in personalities which rendered it impossible to continue the publication under the same management. On p. 174 of the first volume it was stated that "The Health of Mr. Baron, our principal editor, was last summer entirely destroyed by three of the understrappers of the *health committee*," so that he fortunately had to give way for Robert Adrain to assume the position of editor. The circumstances above described explain why, in the preface to Vol. II, Adrain says, "The editor begs leave to assure his friends of science and of man that nothing unbecoming a christian and a gentleman shall be suffered to make its appearance in the work so long as it

shall be under his direction." Without a knowledge of the lack of tact in the editing of Vol. I of a journal devoted chiefly to problem solving the precise reason for the statement would not be apparent.

A little later (1808) there appeared in Philadelphia a short-lived and relatively unimportant journal, *The Analyst or Mathematical Museum* of which only a few numbers were published. It is not to be confused with *The Analyst* which Robert Adrain edited and which began a more distinguished career in New York in 1814, or with the periodical by the same name edited from 1874 to 1884 by J. E. Hendricks. Although Adrain's *Analyst* was of the problem-solving type, it was edited by a man of recognized mathematical ability and it published numerous articles or solutions by such scholars as John Gummere, F. R. Hassler, Robert Patterson, Nathaniel Bowditch, and Adrain himself.

Somewhat similar to *The Mathematical Correspondent* was *The Mathematical Diary* (New York, 1825–1832, 13 numbers), and like the former was similar to some of the English periodicals of the eighteenth century but without their literary or scientific standing. The trivial nature of the problems proposed may be inferred from the one requiring the solution of the equations $x(x+y+z)=6$, $y(x+y+z)=12$, $z(x+y+z)=18$.

In 1816 a publication under the name *The Portico* was begun, and in the second volume a department of mathematical problems was opened. It is mentioned for the reason that men like Robert Adrain and Claude Crozet contributed various problems, generally demanding ingenuity and patience rather than any knowledge of mathematics beyond the elementary field.

A rather curious character came to America early in the century, William Marrat, A.M. (1772–1852), who announced himself as a "Fellow of the New-York Philosophical Society." Before leaving England he had published *The Enquirer, or Literary, mathematical, and philosophical Repository,* a quarterly "conducted by" himself and Pishey Thompson (Boston, England) and printed in London.[3]

In New York Marrat started (1818) *The Monthly Scientific Journal* announcing that "he will never shrink from any labour by which he conceives mankind may be benefited." The nature of the mathematics may be inferred from the fact that the first "mathematical question" proposed was "Prove that $a^0=1$," and the first problem solved was to find the values of $x$ and $y$ from the equations $x+y=a$ and $xy=b$. In the preceding year (1817) Marrat organized a society of mathematicians in New York. It consisted of eight members, including Adrain who, judging by the nature of the periodical, must have felt rather out of his element. Nevertheless Marrat is entitled to consideration because of his efforts to increase the interest, although to a relatively slight degree, in mathematics in the New World. He evidently returned to England after a few years, for in 1825 he seems to have been again editing *The Enquirer* at Liverpool.

About this time there also appeared *The Ladies' and Gentlemen's Diary or United States Almanac* (New York,

[3] Three volumes were published in the years 1811–1813. Thompson (1784–1862) dropped out before Vol. III appeared. Vol. II has no title page (Library of Congress copy), and he may not have assisted in editing that.

1820), edited by Melatiah Nash.[4] It consisted largely of astronomical articles and of mathematical queries. Nash had a school at 331 Broadway, probably the best for the study of advanced mathematics at that time in New York.

Among the problem-solving periodicals mention should be made of the *Mathematical Miscellany* published by Charles Gill of the Flushing (N. Y.) Institute, which appeared semi-annually for the four years of 1836–1839. It had the merit of offering more advanced material than most others of its type. Gill's personal contributions are mentioned later in this chapter.

Passing over the other publications of this kind, we find a distinctly higher standard set when, in 1842, Benjamin Peirce and Joseph Lovering, both on the Harvard faculty, started *The Cambridge Miscellany of Mathematics, Physics and Astronomy*, intended as a quarterly. The editors enlisted the cooperation of a number of contributors of ability, but the periodical met the needs of neither the high-school teachers nor the college men and was far from having university standing, and hence it was short-lived.

A journal which gave promise of furnishing an opportunity for the publishing of articles which, for the period, were of superior quality was founded in 1858 by John Daniel Runkle—the *Mathematical Monthly*, more commonly known as Runkle's Monthly. It lasted, however, only three years.

The other mathematical journals which it seems desirable to include were established in the last quarter of

[4] On the early journals of this type see *Scripta Mathematica*, vol. I, No. 4; New York, 1933.

the century and therefore will be discussed in Chapter IV. Such additional names as should be considered and for which we have space will also be included in that chapter. The majority of these mathematicians completed their work after the year 1875 and may properly be considered in connection with the generation of those who laid the foundations for our present work in this field. The earlier ones, few in number, although frequently the teachers of those whose work was done in the late decades, are included in the present chapter.

The difficulty which scholars had in those early days in finding any way of publishing their scientific articles is seen in the case of William Ferrel (1817–1891), who was "the pioneer in the work of bringing the general character of the circulation of the atmosphere and the meteorological consequences thereof within the domain of mathematical researches . . . It is a curious though lamentable circumstance . . . that a man who had mastered the *Principia* and the *Mécanique Céleste* and who had laid the foundation of the theory of the circulation of the atmosphere should have found no better medium for the publication of his researches than the semi-popular columns of a journal devoted to medicine and surgery."[5]

It should be added, however, that Ferrel's later memoirs were more favored, appearing in Runkle's

[5] R. S. Woodward, "The century's progress in applied mathematics," *Bull. Am. M. S.*, VI (2), 133–163. The article referred to was his "Essay on the winds and the currents of the ocean." It was published in the *Nashville Journal of Medicine and Surgery* in 1856. Cleveland Abbe (published a biographical sketch of Ferrel in the *Bull. Washington Phil. Soc.*, XII, 448–460. See also *The American Meteorological Journal*, VIII, 241–244.

*Monthly* (1859, 1860) and in the *Report of the Superintendent of the U. S. Coast and Geodetic Survey* (1875, appendix 20, pp. 370–412; 1878, appendix 10, pp. 176–267; 1881, appendix 10, pp. 225–268, 337–369).

Newcomb called attention to the fact that Ferrel "was the first person to show, from correct theory, that the action of the moon in causing the tides should produce a retardation of the earth's rotation." Ferrel's article appeared in Gould's *Astronomical Journal* in 1853. Newcomb further remarks: "As frequently happens in the history of science, the first discoverer in a new field has himself to be discovered by antiquarian research."

## 4. PROMINENT NAMES, 1800–1875

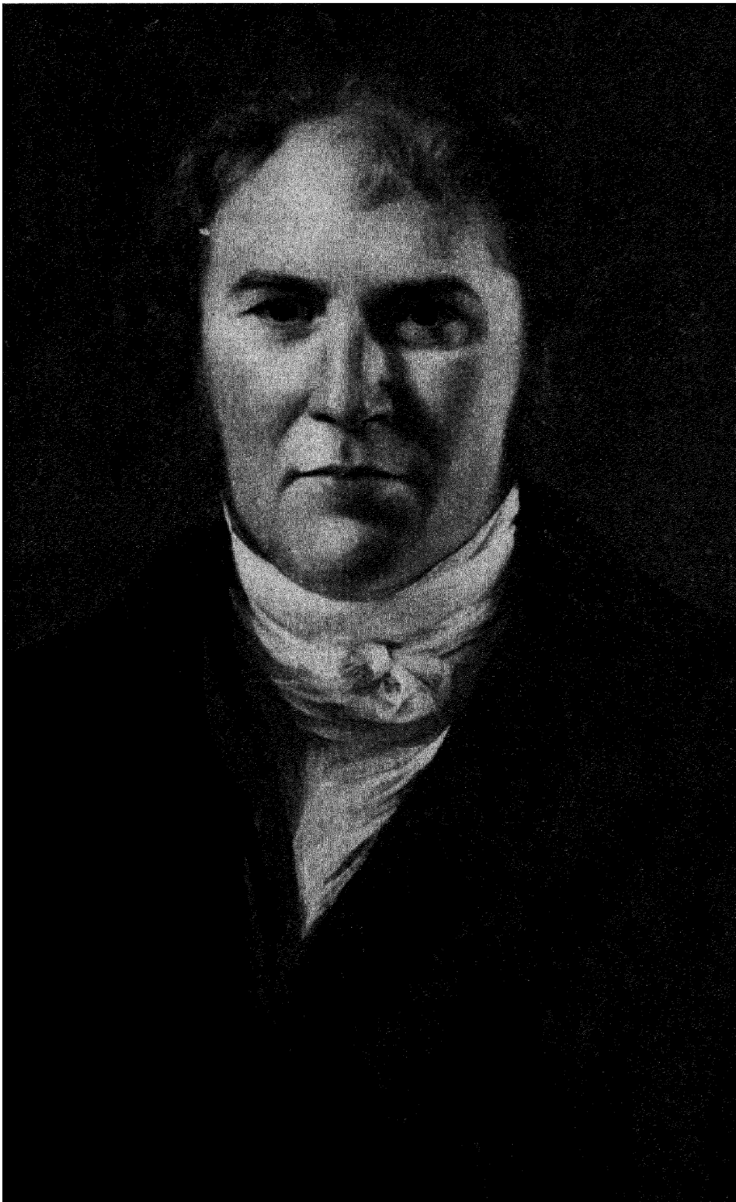
In the first seven or eight decades of the nineteenth century only a few American scholars made any noteworthy contributions to what would now be called the domain of higher mathematics. Most of those who are generally recognized as having made such contributions died after the year 1875 and are considered in the next chapter. A few of the prominent names of those who died before that date will suffice to show the nature of the mathematics of the first three quarters of the century.

Not considering those scholars whose chief interest was astronomy, we think it is safe to say that the nearest approach to a mathematican of the first class in America at the opening of the nineteenth century was **Robert Adrain.** He was born in Ireland in 1775, came to America in 1798, was connected with the University of Pennsylvania, Princeton, Rutgers, and Columbia, and

died in 1843. He founded *The Analyst* (1808) and the *Mathematical Diary* (1825) as already mentioned. In 1812 he edited Charles Hutton's *Course of Mathematics*, making various corrections, not always for the better, and to the fifth American edition (1831) he added a supplement on descriptive geometry (pp. 579–640). Of original contributions his proof of the law of least squares, his investigations relating to the figure of the earth and its mean diameter, and his study of the force of gravity at different latitudes are the most important. Those on the study of the earth appeared in the publications of the American Philosophical Society. He also edited the *Mathematical Correspondent*.

One of the most interesting pieces of translating and editing done by an American mathematician of this period was that of Nathaniel Bowditch[1] on the *Mécanique Céleste* of Laplace, already mentioned. Largely a self-taught mathematician, Bowditch not only translated the work but added a large number of explanatory notes necessary to elucidate the meaning of the author. The translation was made in the years 1815–1817 and was not completed. Four volumes appeared in Boston in 1829–1839, the proofs being revised and corrected by Benjamin Peirce, then in his twenties. Two other volumes were promised in case there was sufficient demand to warrant their publication,—a demand which America was not prepared to meet. Between the time of the translation and the publication of the first volume the English translation of Volume I by J. Toplis (Nottingham, 1814) appeared, and certain notes in this work were used by Bowditch. During the printing another

[1] b. Salem, Mass., March 26, 1773; d. Boston, March 16, 1838.



Robert Adrain (1775–1843)

English translation, by H. H. Harte, was also in process of publication (2 vols., Dublin, 1830). Of Bowditch's translation of Laplace Sir John Herschel wrote in the *Quarterly Review* that it was noteworthy as the first great scientific work issued in the United States and that it left no step in the text unsupplied and hardly any material difficulty either of conception or of reasoning unelucidated. To Bowditch himself Herschel wrote (March 8, 1830) these words of praise:

> It is a work, indeed, of which your nation may well be proud, as demonstrating that the spirit of energy and enterprise which forms the distinguishing feature of its character, is carried into the region of science; and every expectation of future success may be justified by such a beginning.

As to his scholarship, it is related that he began the study of Latin at the age of seventeen for the purpose of being able to read Newton's *Principia*, and later became proficient in French, German, Italian, and Spanish so as to read the astronomical treatises in these languages. His *New American Practical Navigator* appeared in 1802 and was well received, but it was his translation of Laplace that gave him an international reputation.

Bowditch's translation led Robert Adrain to write in *The Mathematical Diary* (II, 298), "There has been a fluttering among the stooled pigeons of Europe at this achievement of our countryman; and the automaton professors there begin to open their eyes at last to see what has been snatched from them."

In the *Encyklopädie der Mathematischen Wissenschaften* (VI, part 1, p. 235) mention is made of Bow-

ditch's attempt at computing a non-elliptical meridian by a method analogous to one used later by M. G. von Paucker who had assumed the radius of curvature to be given by the formula[2]

$$\rho = a(1 + \alpha \sin^2 \phi + \beta \sin^4 \phi + \gamma \sin^6 \phi + \delta \sin^8 \phi).$$

Bowditch was, however, not merely an astronomer. The first work in America which showed much originality in the field of geometry was his study of a class of curves defined by the equations $x = a \sin(nt+c)$, $y = b \sin t$. In his investigations of the apparent motion of the earth as seen from the moon, Bowditch found that a study of these curves for special cases was necessary. His results were published in 1815 in the *Memoirs of the American Academy of Arts and Sciences* (vol. III, part II, pp. 413–436). Many years later, in his study of certain phenomena in acoustics, the French physicist Jules-Antoine Lissajous (1822–1880) had occasion to investigate them, and hence in Europe they are generally known as the Lissajous curves. Analogous curves in three-dimensional space were studied late in the century (1898) by Elting Houghtaling Comstock (*b.* 1876) and Charles Sumner Slichter (*b.* 1864), the results appearing in vol. XII of the *Transactions of the Wisconsin Academy*. After Bowditch the next prominent mathematician to give attention to mathematical curves was Benjamin Peirce whose work will be considered later in Chapter IV.

[2] F. R. Helmert, *Die Mathematischen und Physikalischen Theorien der Höheren Geodäsie* (Leipzig, 1884, 2 vols., I, 17, 18; II, 15), where the methods of von Paucker, Clarke, and Ritter are mentioned. See also *The ordnance Trigonometrical Survey; Principal Triangulation* (London, 1858).

Among the contemporaries of Bowditch and doing much less than he for the advancement of his subject, was **Robert Patterson** (1743–1824), professor of mathematics in the University of Pennsylvania (1782–1813) and vice-provost (1810–1813). He was succeeded in both positions (until 1828) by his son, Robert Maskell Patterson (1787–1854). Neither one contributed to mathematical research, but the former edited various works, including one on the philosophy of Newton, the Simpson *Euclid* and *Algebra*, and the lectures of James Ferguson (1710–1776) on mechanics, hydrostatics, astronomy, and allied subjects.

Contemporary with the Pattersons was a man of similar abilities, **William B. Rogers** (1804–1882), professor of mathematics and natural philosophy at William and Mary College, where he succeeded his father in the same chair. He later (1835) succeeded the younger Patterson in the chair of natural philosophy at the University of Virginia (1835–1853) and in 1865 founded and became the first president of the Massachusetts Institute of Technology.

Mention has already been made of the translations from the French in the first quarter of the nineteenth century, and of the prominent part taken by **John Farrar** (1779–1853). He was graduated at Harvard in 1803, receiving the A.M. degree in 1805. Two years later he became Hollis professor of mathematics and natural philosophy, holding this position until 1836. His interests were largely in astronomy, but he did much for elementary mathematics in this country through his translations (1818–1825) of the works of Euler, Lacroix, Legendre, and Bézout, and through his publication of a

number of textbooks. He was one of the best teachers of his time and his presence on the Harvard faculty was particularly welcome when he was appointed. John Winthrop had given the college high standing, but his two immediate successors had done little to maintain its reputation. Farrar, while not a mathematician of first rank, did much to restore the prestige of the department. The further work done at Harvard is considered in Chapter IV.

Of those who helped in preparing the way for the development of the newer mathematics of the second half of the century a few others may be cited for their success in the teaching of the subject. One of the more prominent was **Theodore Strong** (1790–1869). He was a pupil of Jeremiah Day of Yale, and thereafter taught at Hamilton and Rutgers. The nature of mathematics in America during the second quarter of the century is fairly well represented by his contributions. Matthew Stewart's (1717–1785) theorems on the circle were stated without proof in 1746 and were proved by Strong about 1814, some years after the solution by James Glennie in the *Edinburgh Philosophical Transactions* (1805). A series of other problems by Strong and relating to the circle appeared later in *Silliman's Journal* (vol. II). His "Demonstration of Stewart's properties of the circle"[3] was published in the *Memoirs of the Connecticut Academy of Sciences* (I, 393–412). In his *Algebra* (New York, 1859) he suggested that the elements of the

[3] Since Stewart's theorems do not relate directly to our work, it suffices to say that they may be found in the *Cyclopaedia or Universal Dictionary of Arts, Sciences and Literature*, Abraham Rees, London, 1819, article "Circle."

calculus might profitably be taught in the secondary schools. The book also contained a new treatment of the irreducible case of the cubic, certain improvements in the solution of equations, and a method, which appears to have been new, of using Bernoulli numbers in integration. He was an earnest student of the mathematics of his European contemporaries and wrote numerous articles in the *Mathematical Miscellany* relating to difficult portions of Laplace's *Mécanique Céleste.* Other memoirs and solutions of his appeared in various journals, including the *American Journal of Science*, the *Boston Journal of Philosophy*, the *Mathematical Miscellany*, and the *Proceedings of the American Academy of Arts and Sciences.*

This is not the place to speak at any greater length of the early astronomical observers. Suffice it to say that creditable work was done in the closing years of the eighteenth century by Dr. James Madison at the College of William and Mary and by other astronomers, but the mathematics involved was simply that of the standard English texts. It was not until the middle of the nineteenth century that the United States Government could be led to see the necessity for a national observatory, although James Monroe, when secretary of state in 1812, urged congress to undertake this project.

The name of Charles Gill, who has already been mentioned in connection with the *Mathematical Miscellany*, is almost forgotten in this generation, and yet he was a man of undoubted ability and of considerable influence in promoting the study of mathematics in his day. Born in Yorkshire, England, in 1805, the son of a local shoemaker, at the age of thirteen he went to sea, voyaging

to the West Indies, and three years later found himself in command of a ship which had lost its officers by yellow fever. His skill as a navigator enabled him to bring his boat safely to its destination, and this convinced him that theoretical as well as practical mathematics was within his reach. He therefore began his career as a teacher, devoting his leisure hours for several years to the study of the subject. During this time he contributed to various mathematical journals and when, in 1830, he came to America he was already known as an original thinker. He at once engaged in teaching in secondary schools, his final position being in the Flushing Institute on Long Island, N. Y., a school which developed into a short-lived college. Here he brought out the *Mathematical Miscellany* already mentioned. His lines of major interest were the theory of numbers and actuarial work. As to the former, Professor L. E. Dickson mentions his contributions several times,[4] while Dr. Emory McClintock pays him high tribute in a series of articles under the title "Charles Gill: The first actuary in America."[5] Most of his work in pure mathematics appeared in the solution of intricate problems in the theory of numbers published in such journals as *The Ladies' Diary*, *The Gentleman's Mathematical Companion*, *The Educational Times*, and his own *Miscellany*.

An example of the diversified attention commonly displayed by mathematicians in the first half of the century is seen in the work attempted by **Alexander**

[4] *History of the Theory of Numbers*, I, 353; II, 21, 22, 24, 194, 195, 205, 211, 262, 370, 405, 456, 499, 504, 510, 570, 700.

[5] *Transactions of the Actuarial Society of America*, XIV, 9–16, 211–238; XV, 11–40, 227–270.

**Dallas Bache** (1806–1867). He was the author of several articles on astronomy and surveying in the *Proceedings* and the *Transactions of the American Philosophical Society*, and was interested in applied mathematics in general. He was professor of natural philosophy and chemistry (1828–1841) in the University of Pennsylvania, and became (1843) superintendent of the United States Coast Survey, in which position he made his reputation. As a matter of heredity it may be mentioned that he was a descendant of Benjamin Franklin.

For the first organization of the United States Coast Survey (1807), for the beginning of the work (1816), for its revival after fourteen years (1818–1832) of stagnation, and for determining the United States standards of weights and measures, America is indebted largely to the labors of **Ferdinand Rudolph Hassler** (1770–1843), a Swiss scholar who came to Philadelphia in 1805. Although a mathematician of considerable ability, his genius lay in practical geodesy rather than in the advance of the abstract phase of the subject. His *Analytic Trigonometry, Plane and Spherical* (New York, 1826) was much above the other contemporary textbooks, chiefly in its method of presentation, for as to content it showed little originality. One of his most important contributions was a set of tables (New York, 1844). He also published an arithmetic (New York, 1826), a work of little influence. His insistence upon the highest attainable degree of accuracy of standards, both in the instruments used and in the measurements attempted, was of great value in the initial work of the Coast Survey and of what later developed into the Bureau of Standards. He was a visionary in all matters of economics and

politics, and his life was far from a happy one; but he served his adopted country well. It was a just tribute to his fidelity and his ability that there was placed upon his monument the words, "In the memory of Ferdinand Rudolph Hassler. . . . Having filled with honor, both in his native and adopted country, offices of high trust and responsibility died in Philadelphia . . . in the midst of his labors as superintendent of the United States Coast Survey and Standards of Weights and Measures."[6]

In connection with the Coast and Geodetic Survey and with the Bureau of Standards, mention should be made of the considerable interest in the mathematical problem of the pendulum in the nineteenth century. Of the American mathematicians who gave particular attention to the subject the following were among the most prominent, the references being arranged chronologically:[7]

N. Bowditch, *Memoirs of the American Academy of Arts and Sciences*, vol. III (1815); vol. IV (1818);

Robert Adrain, *Transactions of the American Philosophical Society*, vol. XVI (1818);

John Farrar, *Boston Journal of Philosophy and Arts*, vol. I, (1824); *Bulletin de Férussac*, vol. V (1826);

Simon Newcomb, *Astronomische Nachrichten*, vol. LXXXI (1873);

T. C. Mendenhall, *American Journal of Science*, vol. XXXI (1881).

[6] F. Cajori, *The Chequered Career of Ferdinand Rudolph Hassler*, Boston, 1929.

[7] For a more extensive list see the *Mémoires relatifs à la physique* of the Société française de physique, tome IV, Paris, 1889.

# CHAPTER IV

## THE PERIOD 1875–1900

### 1. INTEREST IN MATHEMATICAL RESEARCH

In the last quarter of the nineteenth century there were various influences which tended to revolutionize the study of mathematics in America. They might all be included in the term "Spirit of the Times," but back of this spirit there were causes which are quite evident to those who lived through the period.

Speaking in a general way and remembering various exceptions we had good native talent in the mathematical field, but we were hidebound as to our curriculum and as to our methods of instruction. Our colleges, even those which adopted the university name, were clinging to the traditional courses, the traditional topics, and the traditional way of teaching. Mathematics lacked vision and virility.

Three potent influences were now making for reform, however—a reform which eventually proved to be revolutionary as far as mathematical research was concerned. The first of these was the vision of men like presidents Gilman and Eliot[1]—visions which materialized in the founding of Johns Hopkins University, the calling of James Joseph Sylvester,[2] to take charge of the

[1] Daniel Coit Gilman (1831–1908), president of Johns Hopkins, 1875–1901; Charles William Eliot (1834–1926), president of Harvard, 1869–1909.

[2] 1814–1897, professor of mathematics at Johns Hopkins from 1877 to 1883.

athematical department at this new seat of learning, ıd the developing of a modern faculty of mathematics . Johns Hopkins, and later of similar faculties elsehere. President Gilman had the good fortune of startg a university *de novo*, with no college traditions to ımper him, and with college graduates for his students. resident Eliot had to expand a traditional college into university, and to develop on a large scale a type of ·iginal research which can hardly be said to have exted in the Harvard of his younger days. He knew from ;perience, however, the stagnation of mathematics. He ıd been an instructor in the subject in the college, and om 1854 to 1863 he had been assistant professor of ıathematics and chemistry. Moreover he had made a ıreful study of education abroad and was in sympathy ith Gilman's plans for advanced study and original inestigation. As to Gilman himself, he had been assistant brarian at Yale for seventeen years, a professor in the heffield Scientific School, and president of the Univerty of California and, like Eliot, had known European ducation at first hand.

Each of these men was determined to create a real niversity in America, one building upon an old foundaon and the other laying his own substructure. Each ad an influence which it is difficult to describe in mod:ate terms and mathematics owes to each a debt that ın hardly be overestimated. The old colleges with uniersity names began to awaken, to secure men who had :udied under the great leaders in Europe, and to en.rely revise their courses in all subjects. Professors who ad no greater qualifications than the ability to teach ıe calculus in high-school fashion and to compute an

eclipse by rule, gave place in natural course to younger and far better trained mathematicians. In this movement strong support was given by such other old foundations as Yale, Columbia, Princeton, and Brown, which showed by their deeds that the traditional state of affairs could no longer endure in a country which was rapidly becoming wealthy enough to support universities of high rank. Education has generally been a kind of fetish in new countries, and so it has been in the United States and Canada. As the nineteenth century drew to a close the state universities began to rival the older semi-private ones of the East, and often to surpass them in equipment and in personnel. The influence of Johns Hopkins, Harvard, Yale, Columbia, Brown, and Princeton was felt in every state and province.

Sylvester's name, his scholarship, and his influence drew to him a small body of earnest workers seeking guidance. In collaboration with W. E. Story he founded in 1878 the *American Journal of Mathematics* as a medium by which to make accessible the papers written by himself, his pupils, and other mathematicians sufficiently trained in America or abroad to offer such original material as he was willing to recognize. In his address on "Mathematical Progress in America," delivered in 1904 as president of the American Mathematical Society, Professor Fiske called attention to the fact that the first ten volumes of that publication contained papers contributed by about ninety different writers, thirty being from foreign countries and about one third of the others being Sylvester's pupils. Among the contributors to the early numbers were such scholars as Simon Newcomb (later to become editor of the

*Journal*), George William Hill, Josiah Willard Gibbs, Charles S. Peirce, Emory McClintock, William Woolsey Johnson, Washington Irving Stringham, Thomas Craig, and Sylvester himself, so that, Minerva-like, the periodical was born fully matured. The work done by these men and by Sylvester and his other followers will be considered later.

## 2. THE AMERICAN MATHEMATICAL SOCIETY

The second great influence in developing the spirit of scientific research in the field of mathematics was the founding of the American Mathematical Society.

Following his graduation from Columbia in 1885, **Thomas Scott Fiske** (*b.* 1865) spent three years there in graduate work, taking his Ph.D. degree in 1888. After acting as tutor for some time he continued his studies at Cambridge University and in Germany, becoming an instructor at Columbia in 1891, adjunct professor in 1894, and professor in 1897. These dates are given as having a bearing upon the founding of the Society. On November 24, 1888, he suggested to a half dozen members of the Department of Mathematics at Columbia that they meet each month to discuss matters of general interest in their fields of activity. A month later, at his suggestion, it was decided that they should consider their group as the nucleus of the New York Mathematical Society, inviting to membership all others in the city and vicinity who might have a professional interest in mathematics. The first president was John Howard VanAmringe (1835–1915), head of the department at Columbia, and Professor Fiske was the first secretary. In December, 1890, Dr. Emory McClintock,

who had come to New York the year before, was elected president. At the same meeting J. K. Rees was elected vice-president, Harold Jacoby treasurer, Thomas S. Fiske secretary, and W. Woolsey Johnson, Daniel A. Murray, James Edward Oliver, and J. H. VanAmringe the other members of the council, with Professors Fiske and Jacoby the committee on publications.

In 1891 it was decided to publish a journal known as the *Bulletin of the New York Mathematical Society*, Professor Fiske being the chief editor. As Secretary he reported that he had received the support of Professors Simon Newcomb, W. Woolsey Johnson, Thomas Craig, and H. B. Fine, both as to the Society itself and as to this publication. As regards the membership of the Society in that year it is of interest to know that fifty held the degree of doctor of philosophy or of science, that eight were members of the London Mathematical Society, and that three were members of the French Mathematical Society. By June of the same year the membership had increased to 174, and a year later to 227, twenty-four being added to the number of doctors and eight to the number who were also members of the London Mathematical Society.

It was becoming evident that the Society was no longer local and that the name failed to represent the scope of its membership and its mathematical interests. In the year 1894 it was therefore decided to change its name to the American Mathematical Society,[1] the first president (the third from the founding of the New York Mathematical Society) being George William Hill. The

[1] Simultaneously the name of the Journal become the *Bulletin of the American Mathematical Society.*

work now began to move even more rapidly. In 1896 the first Colloquium was held, and in the following year the Chicago Section of the Society was established. This was the first of the sections to be formed and was the result of a mathematical conference held at the University of Chicago on December 31, 1896, and the following day, Professor E. H. Moore presiding. A committee on arrangements consisting of H. S. White, E. M. Blake, and J. W. A. Young was appointed for the spring meeting, the second one being held the following December at Northwestern University.[2] Thomas F. Holgate was made Secretary of the Chicago Section, a position which he held for the succeeding ten years.

It had now become evident that the number of important papers prepared by the members rendered necessary an extension of the means of publication. At a meeting of the Council on August 19, 1898, a committee consisting of Professors Fiske, Newcomb, Moore, Bôcher, and Pierpont was appointed "to consider the question of securing improved facilities for the publication of original mathematical articles in this country." The committee reported on October 29 that it was of the opinion that the Society should make an attempt to join with Johns Hopkins University in the publication of the American Journal of Mathematics. A scheme of mutual action was submitted, but on December 28 the committee reported that "their negotiations with Johns Hopkins University had not been successful." On February 25, 1899, the Committee recommended to the Coun-

[2] *Bull. A. M. S.*, III, 199, and IV, 182. These details are mentioned because this was the first of the Sections. The others were formed after 1900.

cil the appointment of a new committee for the securing of "financial guarantees for the undertaking." This was accepted, and the new committee was appointed consisting of Professors Fiske, Woodward, Moore, Bôcher, and Pierpont. On April 29, 1899 this committee reported "that subscriptions of one hundred dollars a year for five years had been practically guaranteed by representatives of each of the following institutions: Bryn Mawr College, Cornell University, Haverford College, Princeton University, Columbia University, Chicago University, Northwestern University, Yale University, Harvard University." The following Editorial Board was appointed: Professors E. H. Moore (to serve until February 1904), E. W. Brown (until 1903), and T. S. Fiske (until 1902). Among the necessary changes in the conduct of the Society was this: "By-Law XI. 1 (To read) The Society shall publish a periodical which shall have for its object to make known as widely as possible the more important researches presented at the meetings of the Society. This periodical[3] shall be called 'The Transactions of the American Mathematical Society.'"

As the result of this action the *Transactions* made its first appearance in January, 1900, with Professor Moore as Editor-in-Chief, a position which he held till 1908.

One of the most important stimuli to American mathematics in the last decade of the nineteenth century was the Chicago Congress of 1893. This was held in connection with the International Exposition commemorating the four hundredth anniversary (1892) of the discovery

[3] On account of the historical importance of this action of the Society the above quotations, taken from the minutes of the Council, have been given for future reference.

of America.[4] To this congress were invited eminent mathematicians from abroad, and among those attending or sending papers were:

Norbert Herz (Vienna), Felix Klein (Göttingen), E. Study (Marburg). The following foreign scholars communicated papers: H. Burkhardt (Göttingen), A. Capelli (Naples), Walther Dyck (Munich), R. Fricke (Göttingen), Lothar Heffter (Giessen) Charles Hermite (Paris), David Hilbert (Königsberg), Adolf Hurwitz (Göttingen), Felix Klein (Göttingen), Martin Krause (Dresden), Emile Lemoine (Paris), Matyáš Lerch (Prag), Franz Meyer (Clausthal), H. Minkowski (Bonn), E. Netto (Giessen), Max Noether (Erlangen), Maurice d' Ocagne (Paris), Bernard Paladini (Pisa), T. M. Pervouchine (Kasan), Salvatore Pincherle (Bologna), Alfred Pringsheim (Munich), Victor Schlegel (Hagen), Arthur Schoenflies (Göttingen), E. Study (Marburg), Heinrich Weber (Göttingen), Edouard Weyr (Prag).

The list is given in full in order to show the predominating influence exerted upon the work in this country by certain mathematical centers of Europe and by certain prominent scholars. The following thirteen scholars resident in America presented papers:

Oskar Bolza (Chicago, Ill.), Frank N. Cole (Ann Arbor, Mich.), William H. Echols (Charlottesville, Va.), Henry T. Eddy (Terre-Haute, Ind.), George B. Halsted (Austin, Texas), Alexander Macfarlane (Austin, Texas), Artemas Martin (Washington, D. C.), Heinrich Maschke (Chicago, Ill.), Eliakim H. Moore (Chicago, Ill.), Joseph de Perrott (Worcester, Mass.), Albert M. Sawin (Evansville, Wis.), Washington Irving Stringham (Berkeley, Calif.), Henry Taber (Worcester, Mass.).

The officers of the Congress, here mentioned for purposes of record, were as follows: Honorary President,

[4] *Mathematical Papers read at the International Mathematical Congress held in . . . Chicago* 1893." Edited by E. H. Moore, Oskar Bolza, Heinrich Maschke, H. S. White, New York, 1896.

Felix Klein; President, W. E. Story; Secretary, H. W. Tyler; Executive Committee, Professors Story, Tyler, Klein, and H. S. White.

After the Congress adjourned Professor Klein consented to hold a colloquium on mathematics, with such members as might wish to participate. This was held at the Northwestern University, Evanston, Ill., from August 28 to September 9, 1893, during which period Klein delivered a series of lectures which were reported by Professor Alexander Ziwet of the University of Michigan and which were published in 1893 (2d edn. 1911).

As a matter of record the following list of presidents, vice presidents, and secretaries of the American Mathematical Society, elected before 1901, and dating from the founding of the New York Mathematical Society, is inserted:

Presidents. J. H. VanAmringe (1888–1890), Emory McClintock (1890–1894), G. W. Hill (1894–1896), Simon Newcomb (1896–1898), R. S. Woodward (1898–1900).

Vice Presidents. Emory McClintock (1889–1890), J. K. Rees (1890–1891), H. B. Fine (1891–1893; president, 1911–1912), G. W. Hill (1893–1894), Hubert A. Newton (1894–1896), R. S. Woodward (1896–1898), E. H. Moore (1897–1900; president, 1901–1902), T. S. Fiske (1898–1901; president, 1903–1904), H. S. White (1900–1901; president, 1907–1908).

Secretaries. T. S. Fiske (1888–1895), F. N. Cole (1895–1926).

In the same period the Committee on Publications, important in any such society, included at various times the following members: T. S. Fiske, Harold Jacoby, Alexander Ziwet, Frank Morley, F. N. Cole.

This brief sketch brings the history of the Society up to the close of the period under consideration. It should be

added, however, that the Society began the publication of its Colloquium lectures in 1905 and that this has been continued at varying intervals; that its library has increased from 121 volumes, at the time when a systematic effort was first made to enlarge it, to more than 7700 in 1934.

When it is considered that in 1888 the nucleus of the American Mathematical Society consisted of six members, and that this was the only association of mathematicians in that period which has since developed into a national organization, the enrollment forty-five years later in the three mathematical societies of the country is impressive. The American Mathematical Society in 1934 numbered approximately 1800 members; the Mathematical Association of America, 2000; and the National Council of Mathematics Teachers, a secondary-school organization, 4000.

It is difficult to calculate the relative influence of the American Mathematical Society and of the universities in the development of mathematics in America before 1900, and in the remarkable progress made in the twentieth century. It is, however, safe to say that without both of these agencies the United States and Canada could not have attained their present position in the mathematical field.

## 3. EUROPEAN INFLUENCE

The third great influence in giving America a worthy place in the mathematical world was the result of our contact with European scholars. In the eighteenth century it was not unusual for the sons of the wealthy colonists to go abroad for study in the universities, but

this did not noticeably affect the mathematical situation. The young men went to Europe for general culture, chiefly to England for social reasons or else to Paris for the purpose of acquiring the French language. In the nineteenth century Germany began to take the lead in university work, and in the last quarter of the century students wishing to enter the field of advanced mathematical study in continental Europe generally preferred Göttingen, Leipzig, Berlin, or other such scholastic centers, to Paris or the Italian university cities.

So marked was this tendency that Professor Fiske, in his presidential address of 1904, asserted that probably about 10% of the members of the American Mathematical Society had received the doctorate from German universities, and that 20% had for some time at least pursued mathematical studies in Germany. This was relatively a large number, a much larger percentage than would now (1934) be the case, the lower rate being due chiefly to the fact that the facilities in America have so greatly improved as to render a European training in mathematics less important.

There is a tendency to attribute the sudden outburst of mathematical energy in America in the period 1880–1900 to the presence at Johns Hopkins of Sylvester and, for a brief period, of Cayley, and indeed much is due to their efforts. The latter was, however, a poor lecturer, and the former exercised his influence rather through his suggestions to individual students and through his published memoirs than through his skill in exposition before an audience. Neither one was a Felix Klein. Each opened up lines of research that were relatively new to the students who came under his influence, but neither

one created a school of mathematics commensurate with that of Göttingen, Berlin, or Leipzig. Indeed, it is a question whether it was Johns Hopkins that is chiefly responsible for the awakening of America to its possibilities in the domain of mathematics, or that earnest group of graduates of Harvard, Yale, and other univerties in this country who, at this time, went to Germany and returned with a new zeal for research in fields of which they had not been aware.

In order to form some idea of the German influence upon the mathematics of America in the period under discussion, the following list has been prepared, giving the names of members elected to the American Mathematical Society before 1901 and who are reported as having received the doctor's degree in German or Austrian universities. It is compiled from the list of members published in the *Annual Register* of the Society in 1903 and includes only the names of those who report the universities which conferred the degrees. It does not include the considerable number of names of those who took the degree in American universities and who afterwards studied in German Universities, there being no complete list of such names available. It should not be taken as a record of all who are known to the authors as having taken such degrees or pursued such studies, since any such attempt would depend upon personal recollection and would be open to proper criticism. The following is the list:

Benton, J. R. (Göttingen); Benner, H. (Erlangen); Blichfeldt, H. F. (Leipzig); Bôcher, M. (Göttingen); Bolza, O. (Göttingen); Bouton, C. L. (Leipzig); Chittenden, J. B. (Königsberg); Fine, H. B. (Leipzig); Foche, Anne B. (Göttingen); Hancock, H. (Ber-

lin); Haskell, M. W. (Göttingen); Laves, K. (Berlin); Leuschner, A. O. (Berlin); Lovett, E. O. (Leipzig); Magie, W. F. (Berlin); Manchester, J. E. (Tübingen); Mann, C. R. (Berlin); Maschke, H. (Göttingen); Newson, Mary W. (Göttingen); Noble, C. A. (Göttingen); Osgood, W. F. (Erlangen); Page, J. M. (Leipzig); Peck, H. A. (Strassburg); Peirce, B. O. (Leipzig); Pierpont, J. (Vienna); Pupin, M. I. (Berlin); Reid, L. W. (Göttingen); Roe, E. D. (Erlangen); Rothrock, D. A. (Leipzig); See, T. J. J. (Berlin); Smith, W. B. (Göttingen); Snyder, V. (Göttingen); Story, W. E. (Leipzig); Townsend, E. J. (Göttingen); Tyler, H. W. (Erlangen); Van Vleck, E. B. (Göttingen); Webster, A. G. (Berlin); White, H. S. (Göttingen); Wilczynski, E. J. (Berlin); Woods, F. S. (Göttingen).

Partly by way of comparison of the influence of European countries, the following list, compiled in the same way and from the same source, gives the names of those who are known to have taught in America and who received a European degree equivalent to the German Ph.D.:

Brown, E. W. (D. Sc., Cambridge); Conoscente, E. (Math. D., Palermo); Macfarlane, A. (D. Sc., Edinburgh); Morley, F. (D. Sc., Cambridge); Saurel, P. L. (D. Sc., Bordeaux); Scott, Charlotte A. (D. Sc., London).

## 4. PERIODICALS

In the preceding chapter mention was made of the most important mathematical journals of the period 1800–1875. In the succeeding quarter of a century the problem-solving style of periodical was much less in evidence. The New World had outgrown the age of mere puzzles of an elementary type and was beginning to demand problem solving of a more advanced nature and articles showing evidence of originality in a new and more fertile field. As already stated, The Mathematical

Society began to publish a journal of much higher rank than those of a half century earlier and other important periodicals began to appear, and these deserve at least brief mention.

The standard attempted by Peirce and Lovering was now more successfully maintained by a man of much less ability, but one having a natural gift in geometry, the theory of numbers, and algebra—Artemas Martin (1835–1918). He published (setting the type and making the wood-cuts himself) the *Mathematical Magazine* (Erie, Pa., vol. I, 1882–1884, 12 numbers)[1], and the *Mathematical Visitor* (Erie, Pa. and Washington, D. C., Vol. I and Vol. II to No. 4, 1877–1894); and the problems and papers which appeared were on the whole the best that had been seen in American journals of this kind. With J. M. Greenwood he wrote *Notes on the History of American Text-books on Arithmetic* (Washington, 1900), a bibliography by no means complete. His line of interest may be seen in his correction of Barlow's minimum solution of the equation $x^2-5658y^2=1$, giving as his solution $x=1284836351$ and $y=17081120$.[2] Professor Dickson has pointed out however, that Barlow's 5658 is a misprint for 56587.[3]

The best-known of this earlier type of mathematical

[1] Martin was appointed librarian of the U. S. Coast and Geodetic Survey in November 1895 and moved from Erie to Washington. There seems to have been a Vol. II, but only isolated numbers are known to the authors of this book. The number dated 1884 was printed later, probably in 1887, this being the date on the title page of the completed volume.

[2] *Analyst*, II, 140. For other typical articles see *ibid.*, IV, 137, 154, 175; *Messenger*, VII (2), 24–26, 50–57.

[3] *Hist. Theory of Numbers*, II, 382. See also R. Garver, "The Analyst," *Scripta Mathematica*, I, 324, probably the best study of any American journal of mathematics that has yet appeared.

periodical was *The Analyst* published at Des Moines, Iowa, from 1874 to 1883, by **Joel E. Hendricks** (1818–1893), a man with a natural taste for mathematics, but with no mathematical training of the university type. He was so skilled in his editorial management, however, that he secured a considerable number of contributors and, for a time, the periodical was one of the few media for the publication of mathematical papers in this country. It began in 1874 as a 16-page monthly, but in 1876 it changed to thirty-two pages and appeared every two months. As to its high standard for the time, Dr. Raymond Garver, in his critical study of the periodical and its influence mentioned above, has called attention to the fact that the *Jahrbuch über die Fortschritte der Mathematik* has 178 references of three or more lines each to articles in *The Analyst*, 28 of them running from ten lines to more than a page. He also gives a list of prominent articles by E. L. de Forest, R. S. Woodward, W. W. Johnson, Sir Thomas Muir, W. E. Heal, Thomas Craig, L. W. Meech, C. H. Kummell, Werner Stille, G. W. Hill, Artemas Martin, and others, with a summary of certain of the most important papers.

After a year devoted to the consideration of continuing the journal, it was decided in 1884 to carry on the work by means of a new publication, the *Annals of Mathematics*. This periodical began its career at the University of Virginia under the editorship of **Ormond Stone**,[4] and was subsequently published at Harvard and later, as now, at Princeton.

[4] B. 1847, d. 1933; professor of astronomy at the University of Virginia; editor of the *Annals* for twelve years (1884–1896) and cooperating editor until his death. His interests were chiefly in astronomy.

Somewhat more successful than others of its type was the *Mathematical Messenger* (1887–1895), edited by G. H. Harvill and devoted largely to problem-solving. The nine years of its existence was an unusual record for a journal of this special kind.

A year after the *Messenger* was discontinued Professor William Edward Story of Clark University began the publication of *The Mathematical Review*, but in spite of its high ideals only three numbers appeared.

*The American Mathematical Monthly* was founded in 1894 by Benjamin F. Finkel of Drury College, with John M. Colaw of Monterey, Va., as co-editor, and began its publication in 1894. The editors were successful in securing early contributions from men of standing in the field of mathematical research, and in 1902 Professor Leonard Eugene Dickson replaced Mr. Colaw as co-editor. A year after the founding of the Mathematical Association of America (1916), largely through the efforts of Professor H. E. Slaught, the *Monthly* was made its official organ.

There is not, in this monograph, sufficient space for considering at any length the publication of papers before the founding of the mathematical societies and periodicals of the last quarter of the nineteenth century. There were, however, several possible means for such publication, chiefly through general scientific journals. This is seen in numerous cases like that of John Nelson Stockwell's paper on symmetric functions, which appeared in *Gould's Astronomical Journal* (1860, VI, 145–149). Stockwell (1832–1920) was a self-taught astronomer and was connected at different times with the United States Coast and Geodetic Survey, the Naval

Observatory, and the Case School of Applied Science at Cleveland. Twenty-seven of his articles appeared in Gould's *Journal* and others were published in the *Observatory* (London) and elsewhere.

In the last quarter of the nineteenth century, needless to say, the interest in American mathematics showed itself largely in the articles published in the periodicals already mentioned—the *American Journal of Mathematics*, the *Bulletin of the American Mathematical Society*, *The Analyst*, the *Annals of Mathematics*, and the *American Mathematical Monthly*.

It must not be forgotten, however, that American scholars were still deeply indebted to British journals for the opportunity of publishing their articles in the English language before their own periodicals were well established. For example, Asaph Hall and W. W. Johnson had several papers in the *Messenger of Mathematics* in the years 1873–1875, and the *Proceedings of the London Mathematical Society*, beginning in volume XI (1879–80), published a considerable number of memoirs by Craig, Ely, Johnson, Taber, Story, Metzler, Miller, Moore, Morley, and Dickson in the last two decades of the century. A selected list of such articles in foreign journals is given later.

## 5. PROMINENT NAMES AND SPECIAL INTERESTS

It is difficult to select a small number of names as the most prominent or the most important in any human activity, in any country, or in any given period. No dividing line exists between great achievements and near-great ones, or between periods. The list of mathematicians given in the *Dictionary of American Biography*

was necessarily limited as to the number of names and the space allowed to each and hence it meets the needs of the general reader rather than those of the historians of mathematics.

The following brief roll, as well as the one given in Chapter III, includes the names of some of the more prominent contributors to American mathematical literature in the nineteenth century, a few of whose works we mention later or which are referred to in standard treatises, and who for the most part were not living at the time this book was written. Since the index provides an alphabetical arrangement of names, the chronological sequence has been continued here as showing more clearly the historical development of mathematics. In general the list is based upon what seemed to be the most important or most generally recognized achievements of the person named and is not intended to give a complete record of his writings.

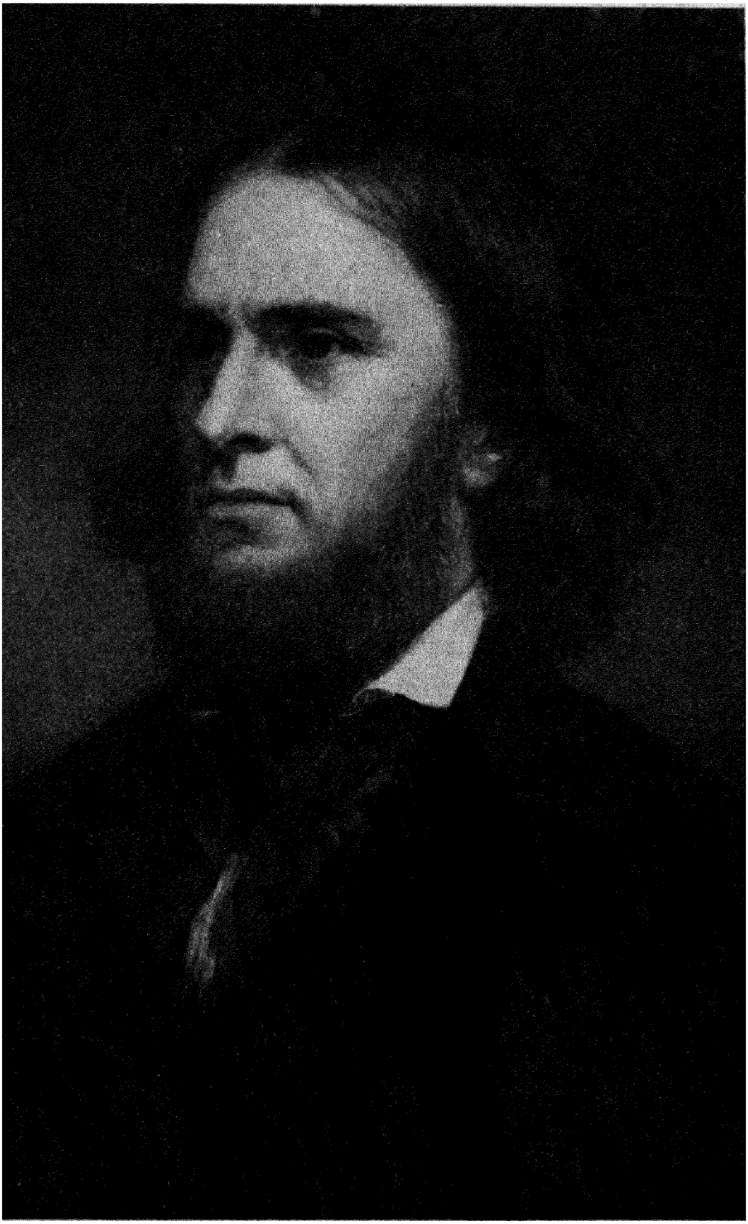
Until the closing years of his life **Benjamin Peirce** (1809–1880) was generally regarded as the leading mathematician, or at least one of the most prominent mathematicians, of America. He was graduated at Harvard (1829) when only twenty years of age and became a tutor there in 1831. He was soon thereafter awarded (1833) the professorship in mathematics and natural philosophy, and later (1842) the professorship in mathematics and astronomy. His outstanding mathematical work was the *Linear Associative Algebra* (1870), of which a new edition (1881) was published in the *American Journal of Mathematics* (vol. IV, reprinted in 1882), edited by his son C. S. Peirce. It ranks as one of the few noteworthy achievements in the field of mathe-

matics in America before the last quarter of the century. In R. C. Archibald's biography (*Benjamin Peirce 1809–1880, Biographical Sketch and Bibliography*, 1925) attention is called to the nature of Peirce's contributions, and especially to the following items: An original and notable result regarding perfect numbers; Methods of determining the number of real roots of equations, applicable to transcendental as well as to algebraic equations; An important advance in the treatment of Kirkman's "Schoolgirl problem," referred to by Sylvester as "the latest and probably the best that had appeared"; A discussion of a new binary system of arithmetic; and An extension of Lagrange's theorem on the development of functions. Of his publications, about 75% were in the fields of astronomy, geodesy, and mechanics.

In his *Synopsis of Linear Associative Algebra* Professor James Byrnie Shaw speaks of Peirce's work on the subject as "really epoch-making." He discusses the adverse criticisms of certain European writers, asserting that they are "due in part to a misunderstanding of Peirce's definitions, in part to the fact that certain of Peirce's principles of classification are entirely arbitrary and quite distinct in statement from those used by Study and Scheffers, in part to Peirce's vague and in some cases unsatisfactory proofs, and finally to the extreme point of view from which his memoir sprang, namely a philosophic study of the laws of algebraic operation."

Dean Herbert E. Hawkes[1] (Columbia) expresses the view that Peirce "aimed to develop so much of the the-

[1] *Am. J. M.*, XXIV, 87–95; *Trans. Am. M.S.*, III, 312–330. Quoted, as also the preceding paragraphs from Archibald, *loc. cit.*



Benjamin Peirce (1809–1880)

ory of hypercomplex numbers as would enable him to enumerate all inequivalent, pure, non-reciprocal number systems in less than seven units. The relation to the problem treated by Scheffers is plain if we remember that the first two of Peirce's principles of classification are identical with those of Scheffers, and the other three are only slightly modified. Peirce solved his problem completely. The theorems stated by him are in every case true, though in some cases his proofs are invalid."

Professor Archibald adds: "Hawkes showed also that by using Peirce's principles as a foundation we can deduce a 'method more powerful than those hitherto given' by such writers as Study and Scheffers for enumerating all number systems of the type considered by Scheffers."

Peirce was one of the earliest champions of quaternions, believing the theory destined to become an important branch of mathematics and not living to see its decline after the introduction of vector analysis.

Among his pupils at Harvard was **Abbott Lawrence Lowell** (*b.* 1856), who was graduated in 1877 with highest honors in mathematics, and who later became president of the university (1909–1933). He showed his interest in the subject by publishing a paper on "Surfaces of the second order as treated by quaternions."[2] Thomas Hill (1818–1891) was also one of Peirce's pupils and became president of Harvard in 1863.

Some idea of the status of the theory of functions in Peirce's time may be derived from an incident related by his students. Having established the relation $e^{\pi/2} = \sqrt[i]{i}$, which had been well known for more than a century, he

[2] *Proc. Am. Acad.*, V (n.s.), 222–251.

remarked, "Gentlemen, that is surely true, it is absolutely paradoxical, we cannot understand it, and we don't know what it means, but we have proved it, and therefore we know it must be the truth."

In spite of his limitations he had a vision of a real university, and Professor J. L. Coolidge has pointed out the fact that before Peirce's time it never occurred to anyone that mathematical research was one of the things for which a mathematical department existed.[3]

The Peirce family was closely associated with Harvard, or with mathematics elsewhere, for nearly a century. Benjamin Peirce's eldest son was **James Mills Peirce** (1834–1906) who was assistant professor of mathematics at Harvard for eight years (1861–1869) and professor from 1869 to 1906. His work was chiefly in the field of applied mathematics. Benjamin Peirce's second son, **Charles [Santiago] Sanders Peirce** (1839–1914) was a man whom Sylvester looked upon as having exceptional mathematical ability, but he was one who failed to use that ability in the way which was probably anticipated. He was graduated at Harvard (1859) at the age of twenty, and at Lawrence Scientific School four years later. For a number of years he was on the United States Coast and Geodetic Survey and became well known for his researches in geodesy. In 1880 he lectured on philosophical logic at Johns Hopkins. His later life was given chiefly to mathematical logic and related subjects. Among his early papers to attract attention was one "On the algebra of logic," and one "On the logic of number."[4]

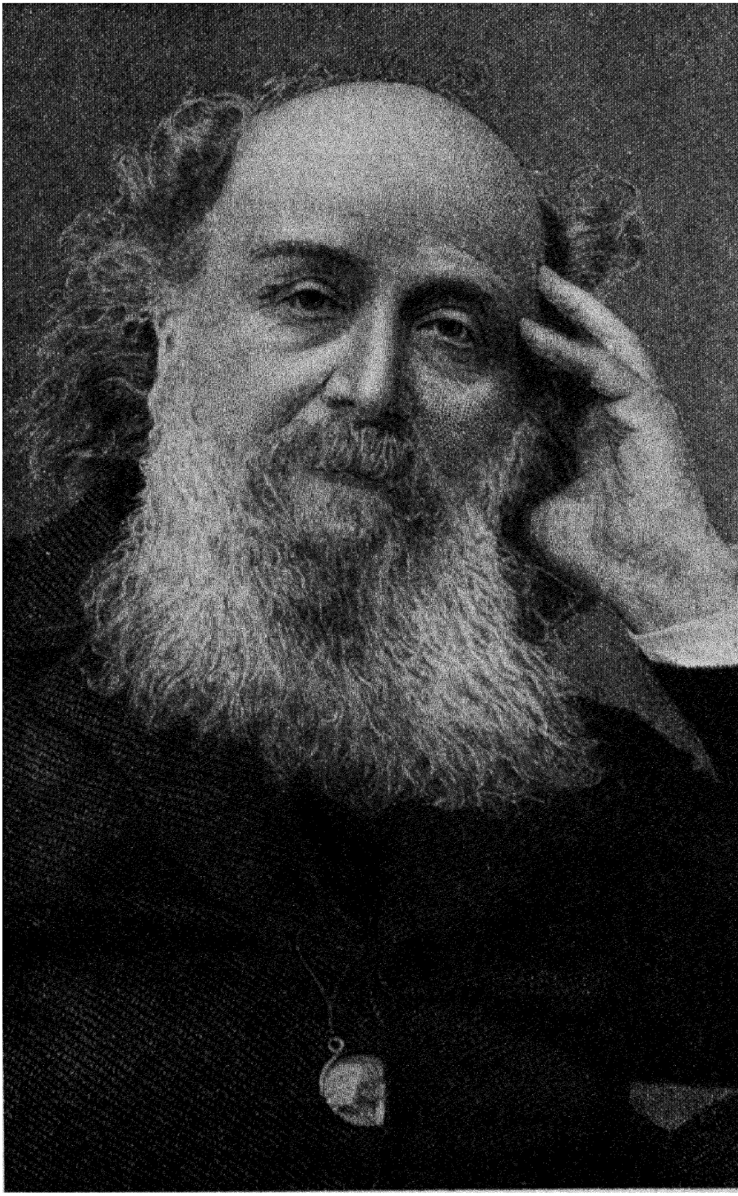
[3] "The Story of Mathematics at Harvard," *Harvard Alumni Bulletin*, XXVI (1924), 376.

[4] *Am. J. M.*, III, 15–57; IV, 85–96; VII, 180–202.

Another relative of Benjamin Peirce, a second cousin once removed, was **Benjamin Osgood Peirce** (1854–1914) whose *Mathematical and Physical Papers 1903–1913* was published by the Harvard Press in 1926. He wrote numerous and valuable papers on mathematical physics, but most of the important ones appeared after the year 1900.

Although not of American birth, and in spite of the fact that he spent only a few years in this country, **James Joseph Sylvester** must be looked upon as one of the greatest leaders in modern mathematical research in the New World. He was born in London in 1814 and died there in 1897. Beginning his education in a boys' school in that city, he completed his preparation for the university in the Royal Institution in Liverpool, having shown unusual mathematical ability in both places. In 1831 he was admitted to St. John's College, Cambridge, and was one of the best scholars of his class. Although he was second senior wrangler in the mathematical tripos in 1837, such was the bigotry of the day that his Jewish faith prevented him from receiving a degree, from competing for the Smith's prize, and from receiving a fellowship. His first academic degree was awarded him four years later by the University of Dublin. It was not until the Tests Act of 1872 that Cambridge could give him any such recognition, but he then received simultaneously the bachelor's and the master's degree.

In 1841 he was called to the chair of mathematics in the University of Virginia, his appointment being in accord with the ideas of Jefferson that European scholars should be drafted to raise the academic standards in this country and particularly in his state. In Sylvester's case,



James Joseph Sylvester (1814–1897)

however, the plan failed, for such was the lack of sympathy between him and his classes that after a few months of continued trouble he was compelled to leave. There are two rare pamphlets, now in the New York Public Library, which seem to have escaped the attention of his biographers. One bears the title, *Testimonials obtained by Prof. Sylvester on occasion of becoming a candidate for the chair of natural philosophy in University College London . . . in the year 1837*, and the other, printed with this, has the title-page *Testimonials obtained by Prof. Sylvester on occasion of offering himself as a candidate for the professorship of mathematics in the University of Virginia in the year 1841*. It is interesting to observe that he is here praised for his success as a teacher, the very thing in which he was a failure.

Sylvester's interest in poetry is well known. He wrote *The Laws of Verse; or, Principles of versification exemplified in metrical translations* (London, 1870), and he printed for private circulation *Spring's Debut, A Town Idyll in two centuries of continuous Rhyme* (*n. d.* but *c.* 1880).

After some experiments on his return to London, in actuarial work and at the bar, he was appointed professor of mathematics in the Military Academy at Woolwich from which position he retired in 1869. In 1877 he was called to Johns Hopkins University for the purpose of building up a worthy graduate school of mathematical research as already mentioned. He retired from this position in 1883 and became Savilian professor of geometry at Oxford. His later years were spent chiefly in London. After leaving Johns Hopkins he continued to contribute to the *American Journal of Mathematics*, his

lectures on the theory of reciprocants, for example, appearing in volumes VIII (pp. 196–260) and IX (pp. 1–37).

In a memorial address on the work of Sylvester, delivered in 1897 by Fabian Franklin, his former pupil and colleague at Johns Hopkins, two particularly significant statements appeared. The first related to his influence upon mathematics in America, and the second to his standing in the mathematical world. Each was characterized by moderation, and each rendered a just decision as to the ability of the man. Referring first to the impetus given to mathematics in America in the last quarter of the nineteenth century Dr. Franklin said: "While there doubtless would, in any case, have been progress in this direction, it must be set down as preeminently the result of Sylvester's presence in Baltimore that mathematical science in America has received the remarkable impetus which the last twenty years have shown."

This states the case with full knowledge of the facts, but with a natural emphasis on the achievements of Dr. Franklin's former colleague. As we have seen, there were other influences, and potent ones, which made for the remarkable awakening of an interest in mathematics in that period, notably the influx of German methods and a knowledge of German achievements; but to Sylvester more than to any other one man is due the remarkable impetus given to the science at that time.

Dr. Franklin's second statement passed judgment with great fairness upon the standing of Sylvester as a mathematician. "In the history of mathematics," he says, "his place will not be with the very greatest: but

his work, brilliant and memorable as it was, affords no true measure of his intellectual greatness." He was, in fact, not a man with the mathematical powers of scholars like Newton, Leibniz, Gauss, Lagrange, Lie, Georg Cantor, or Weierstrass, but to his genius the world owes, for example, much of its interest in the theory of invariants. Building upon the foundation laid by Boole and Cayley, and working with the latter in developing the theory, he was recognized by the entire mathematical world as a leader in this field. Considered more broadly, his work was chiefly in the domain of analysis rather than in that of geometry. In particular he is recognized for his contributions to the fields of the theory of numbers and of higher algebra. In the former he is well known for his Theory of Partitions of Numbers. He also completed Newton's work on the number of imaginary roots in an algebraic equation. As a teacher he was unsystematic and as a lecturer he suffered from this defect. During his few years at Johns Hopkins, he with Professor W. E. Story (1850–1930), founded the *American Journal of Mathematics* to which reference has already been made (see p. 104). His *Collected Works*, in four volumes, were published at Cambridge (1904–1912).

Any consideration of Sylvester's influence upon the study of mathematics in this country would be incomplete without taking into account that exerted by his long-time friend **Arthur Cayley** (1821–1895), even though the latter's sojourn at Johns Hopkins was very brief. He was a graduate of Newton's college (Trinity, Cambridge), senior wrangler, first Smith's prizeman, and both a fellow (1842) and a major fellow (1845). In

1846 he took up the study of the law and for fourteen years continued at the bar. In 1863 he was made Sadlerian professor of mathematics at Cambridge. At the request of Sylvester he was called to Johns Hopkins for a series of lectures on abelian and theta functions during five months in 1882. He was concerned with all branches of mathematics, including mechanics and astronomy, but was known in America chiefly for his work in quantics, matrices, and groups. He contributed twenty-five articles to the *American Journal of Mathematics*, some of which are mentioned elsewhere in this chapter.

Of the astronomers who made noteworthy contributions to pure mathematics **George William Hill**, third president of the American Mathematical Society (1894–1896) ranks among the best known.[5] He was graduated at Rutger's in 1859, and two years later he joined the staff of the Nautical Almanac. Most of his life was given to the study of the lunar theory. On the mathematical side, this led him to the study of differential equations, determinants and series. Professor Moulton[6] has called attention to the fact that the theory of homogeneous linear differential equations with periodic coefficients had its origin in Hill's memoir (1877) on the motion of the moon's perigee. After determining a periodic solution of the differential equations for the moon's motion, and writing out the equations for the variation from the orbit, he was led to a fourth-order system of linear differential equations with periodic coefficients. His work

[5] *b.* Nyack, N. Y., March 3, 1838; *d.* Nyack, Aug. 17, 1916. See article by E. W. Brown, "George William Hill," *Bull. Am. M. S.*, XXI, 499–511.

[6] F. R. Moulton, *Differential Equations*, New York, 1930, Chap. XVII.

also led to the introduction of infinite determinants into mathematics. This memoir[7] was so important that Henri Poincaré was led some nine years later to take up the convergence of infinite determinants, a study continued by H. von Koch. Hill's work on the problem of three bodies occupied his attention for a number of years. Some of his conclusions, as well as such other contributions to the mathematics of astronomy as his memoir "On the development of the perturbative function in periodic series" appeared in *The Analyst.*[8] The one "On the extension of Delaunay's method in the lunar theory to the general problem of planetary motion" first appeared in 1900.[9]

In speaking of Hill's work, Professor E. W. Brown remarks that it exhibits a keen knowledge of theoretical astronomy and the power to handle great masses of numbers. He also calls attention to the influence exerted upon Hill by such writers as Euler, Hansen, and Delaunay, and to the fact that Poincaré's *Les Méthodes Nouvelles de la Mécanique* was based mainly on his idea of the periodic orbit. Concerning a memoir written in 1872 Poincaré says, as noted by Professor Brown, that in it we may perceive the germ of all the progress which has been made in celestial mechanics since its publication.

It is an interesting evidence of the appreciation of Hill's abilities that when Professor R. S. Woodward introduced him to Poincaré the first words that the latter

[7] Reprinted in Hill's *Collected Mathematical Works,* Washington, 4 vols., 1905–1907, II, 243.

[8] II, 161–180. See also his articles in III, 179–185; IV, 97–107.

[9] *Trans. Am. M. S.* for 1900; *Collected Mathematical Works,* IV, 169–206.

said as he took his hand were "You are the one man I came to America to see." This appreciation was shown even more in the introduction which Poincaré wrote to his *Collected Mathematical Works*. He was a contributor to the *American Journal of Mathematics*, to both the *Bulletin* and the *Transactions* of the American Mathematical Society, and to other scientific periodicals in Europe and America. In 1887 the Royal Astronomical Society awarded him its gold medal for his researches in the lunar theory, and he was in receipt of numerous other academic honors. He was an unassuming man of fine character and was a scholar in the best sense of the word.

Primarily an astronomer, **Simon Newcomb** (1835–1909) contributed to mathematics along the same lines as G. W. Hill. Born in Nova Scotia, March 12, 1835, he came to the United States at the age of eighteen and was graduated at the Lawrence Scientific School of Harvard in 1858. He became an assistant in the Nautical Almanac office even before graduation, and after four years of service he became professor of mathematics in the Navy. From 1877 to 1897 he was in charge of the Nautical Almanac office, combining his work there with part-time work as professor of mathematics and astronomy at Johns Hopkins. In 1885 he became editor of the *American Journal of Mathematics*, with Thomas Craig as associate editor. During this period he was also engaged in preparing a new and extensive set of tables of celestial motion. His publications, aside from some elementary textbooks which were not very successful, consisted chiefly of memoirs, tables, and treatises on astronomy. Among his more mathematical memoirs were

those "On the general integrals of planetary motion" (*Smithsonian Contributions to Science*, 1874, pp. 1–31) and on "A generalized theory of the combination of observations so as to obtain the best results" (*Am. J. M.*, VIII, 343–366).

He was the recipient of many honors, including degrees from seventeen universities and numerous medals and prizes from learned bodies representing the field of astronomy. R. C. Archibald (*Memoirs of the National Academy of Sciences*, XVII), in a bibliography of Newcomb, lists 541 titles, and in the same volume W. W. Campbell gives a "Biographical Memoir."

Among the early leaders in the application of mathematics to engineering **John Daniel Runkle** (1822–1902) is deserving of special mention. He was a member of the first graduating class (1851) of the Lawrence Scientific School, receiving both the bachelor's and master's degrees in the same year. Although his major interest was in astronomy, he founded (1858) the *Mathematical Monthly*, a publication of considerable merit, but which, as already stated, continued only three years. In 1849 he began his connection with the *Nautical Almanac* a relationship which continued for thirty-five years. He was professor of mathematics in the Massachusetts Institute of Technology from 1865 to 1902, being acting or official president from 1868 to 1878. His contributions to mathematics were chiefly in his editorial and administrative work.[10]

Among the other mathematical astronomers of the period covered by this chapter was **Asaph Hall** (1828–

[10] H. W. Tyler, *Technology Review*, IV, 277–306.

1907).[11] From 1863 to 1891 he was professor of mathematics at the United States Naval Observatory, and from 1896 to 1901 he was lecturer on celestial mechanics at Harvard. His mathematical work, however, was chiefly in connection with astronomy. He was the discoverer of the satellites of Mars and was a voluminous writer, his published papers numbering more than five hundred. These papers appeared in European as well as American periodicals. For example his memoirs on the rotation of Saturn and on the shadow of a planet were published in the *Astronomische Nachrichten* (LXXXX, 145 and 305). As was the case with the leading mathematicians of the time, he published some of his important papers in *The Analyst*, such as the one on the "Center of Gravity of the apparent disk of a planet" (V, 44) and also one relating to the motion of a satellite (VI, 129).

Several other astronomers of this period deserve special mention because of their mathematical attainments. For example, **James Craig Watson** (1838–1879) in his *Theoretical Astronomy* (Philadelphia, 1869) gave an excellent treatment of the method of least squares and of the application of the theory of probabilities. At the age of eighteen he is said to have mastered the *Mécanique Céleste* of Laplace, and three years later he was made professor of astronomy in the University of Michigan. Before he was twenty-one, as many as forty-seven of his articles had appeared and were later thought worthy of being catalogued in the *Index of Scientific Literature*.

American mathematics is greatly indebted to **Emory McClintock** (1840–1916) for the encouragement given

[11] *Nature*, LXXVII (1907), 154.

to the American Mathematical Society and its publications, and also for his work in actuarial science. He was graduated at Columbia in 1859, and for a year acted as a tutor there. He then spent two years in Paris and Göttingen, studying chemistry and mathematics. After some years in the engineering corps, in banking, and in the consular service he began (1867) his career as an actuary. In this field he became the recognized leader in this country. He was one of the founders of the Actuarial Society of America (1889) and six years later he became its president. As already stated, he was of great assistance in enlarging the purpose of the New York Mathematical Society, of which he was the second president, and in establishing the *Bulletin* and, a few years after the change of name of the Society, the *Transactions*. He wrote a number of important memoirs, but his chief contributions to the literature of pure mathematics were his paper "On the Calculus of Enlargement," *Am. J. M.* (II, 101–161) and his "Analysis of quintic equations" (*ibid.*, VIII, 45–84).[12]

**Washington Irving Stringham** (1847–1909) was graduated at Harvard in 1877, thus beginning his career rather late. From Harvard he went to Johns Hopkins where he took his doctor's degree three years later. He then spent two years under the influence of Klein at Leipzig. Upon his return he entered the faculty of the University of California where he remained the rest of his life. Benjamin Peirce interested him in the theory of quaternions, and in this field he wrote his first contribution in 1878. He was a contributor to the *American*

[12] T. S. Fiske, "Emory McClintock," *Bull. Am. M. S.*, XXIII, 353–357.

*Journal of Mathematics* and other scientific periodicals. Although his contributions were not of the most advanced nature he was influential in making modern mathematics known in this country.[13]

**Robert Simpson Woodward** (1849–1924) was one of the prominent early supporters of the American Mathematical Society and was its fifth president (1899), dating from the organization of the New York Mathematical Society. He was graduated at Michigan (C.E., 1872) and from 1873 to 1882 was assistant engineer on the United States Lake Survey. In 1884 he entered the United States Geological Survey, serving successively as astronomer, geographer, and chief geographer in charge of the division of mathematics. In 1890 he became connected with the U. S. Coast and Geodetic Survey, and three years later was called to Columbia University as professor of mechanics and mathematical physics. From 1905 to 1921 he was president of the Carnegie Institution at Washington. For forty years (1884–1924) he was associate editor of *Science*, and from 1884 to 1889 of the *Annals of Mathematics*. With Mansfield Merriman he edited a series of mathematical monographs, and contributed the one on probability.

The union of mathematics, physics, and astronomy, so common in the eighteenth century and the first part of the nineteenth, was seen much later, as in the case of **Hubert Anson Newton.**[14] Graduating at Yale in 1850,

[13] See Wm. T. Reid and M. W. Haskell in the *University of California Chronicle*, vol. XII.

[14] *b.* March 19, 1830; *d.* Aug. 12, 1896. J. W. Gibbs in *Biographical Memoirs of the National Academy of Sciences*, pp. 101-124, and *Amer. Journ. of Sci.*, III (4), 359–378; A. W. Phillips, *Bull. Am. M. S.*, III (2), 169–173.

he succeeded Anthony Dumond Stanley (1810–1853) three years later as head of the department of mathematics. In 1855 he was given the title of professor with a leave of absence for a year's study under Michel Chasles (1793–1880) in Paris. Although he contributed various articles on mathematics, such as his note on Euler's theorem on the curvature of surfaces,[15] his major interest lay in the fields of astronomy, physics, metrology, and the theory of life insurance. He also published, in 1857, a paper on the gyroscope.

Closely related to astronomy and to the theory of probability is the subject of least squares, already referred to in connection with Robert Adrain. The leading exponent of the subject in America, at least from the historical standpoint, was **Mansfield Merriman.**[16] He was graduated (C.E.) at the Sheffield Scientific School in 1872 and two years later became an assistant in civil engineering at Yale, receiving the degree of Ph.D. in 1876. In 1878 he was appointed professor of civil engineering at Lehigh University. He was editor-in-chief of the *American Civil Engineers' Pocket Book* (New York, 1911), and with R. S. Woodward edited *Higher Mathematics* (New York, 1896). As to the subject of his major interest, he wrote a number of articles upon Least Squares which appeared in *The Analyst* in 1877 (IV, 33–36, 140–143) and compiled a bibliography of works upon the theory.[17] Of his *Elements of the Method of Least Squares* (London, 1877) and *A Textbook on the Method*

[15] *The Analyst*, VII, 93–95.

[16] *b.* March 27, 1848, Southington, Conn.; *d.* June 7, 1925.

[17] *Transactions of the Connecticut Academy*, IV, 151–232.

*of Least Squares* (New York, 1884), the latter served as his dissertation at Yale. He published several textbooks on engineering. He seems to have exhausted the subject of least squares for the time being, for nothing of great importance was published by later writers until after the close of the century. Of the seventy-six articles on the subject listed by J. Howard Gore,[18] seventeen were by Americans. The list of names includes C. Abbe (1871, a history of the method), A. D. Bache (1849, 1854), G. P. Bond (1857), W. Chauvenet (appendix to his *Astronomy*), H. Farquhar (1883), C. H. Kummell (1877, 1880), M. Merriman (1877, 5 items; 1884 and 1894, each with one item), T. H. Safford (1876), C. A. Schott (1855), J. C. Watson (1869), T. W. Wright (1883).

Some idea of the interest in mathematical astronomy in this country and of the interchange of information between America and Germany may be obtained from the articles which our astronomers published in the *Astronomische Nachrichten*. In the following list the numbers refer to the volumes: H. J. Anderson (20), A. D. Bache (18), W. H. C. Bartlett (20), E. Blunt (18), W. C. Bond (8, 9, 18), N. Bowditch (8, 9, 13), and A. Bradley (1, 18).

The name of **William Woolsey Johnson** (1841–1927) occurs frequently in the mathematical periodicals of this period. During forty years he was professor of

[18] A bibliography of geodesy," being appendix 16 to the *Report of the Superintendent of the United States Coast and Geodetic Survey for 1887*, Washington, 1889, pp. 311–512, referred to on p. 42, and containing the list. The second edition appeared as appendix 8 to the Report for 1902, pp. 427–787. It is of historical importance.

mathematics in the United States Naval Academy at Annapolis, Md. In addition to several textbooks he wrote a work entitled *Theory of Errors and Method of Least Squares* (1892) and one on *Differential Equations* (in Merriman and Woodward's *Higher Mathematics*, 1896; published separately a few years later). Some of his papers are mentioned later in the chronological list. They include articles on differential equations (*The Analyst*, IV, 1), curves (*ibid.*, IV, 42), and a "Note on demonstrations of Taylor's Theorem" (*Messenger of Math.*, II (2), 180–184) which led the *Jahrbuch* (V, for 1873) to remark that it combined all the advantages of the then known proofs while possessing none of their disadvantages. To the *Messenger* he contributed several notes on linkages. In the same journal (IV, 58–64) J. W. L. Glaisher discusses the method employed by J. M. Rice and Johnson in obtaining the differentials of functions. In his generalization of the strophoid in Vol. III of the *American Journal of Mathematics* Johnson designates as the generalized strophoid the locus of intersection of two straight lines having two fixed points $A$ and $B$ and rotating in such a way that $n\psi \pm m\phi = \alpha$ where $n$, $m$ are any values (commensurable if the curves are to be algebraic), $\alpha$ is a given angle, and $\phi$ and $\psi$ are the variable angles formed by the fixed line $AB$ and the given lines.

As already stated, the first president of the New York Mathematical Society was **John Howard VanAmringe.** He was born in Philadelphia in 1835 and entered Yale in 1854. After two years he left college to engage in teaching, and in 1858 he entered the junior class of Columbia. In 1866 he was appointed professor of mathe-

matics and in 1894 he became dean of the college. Although he revised the translation of Legendre's geometry he made no important contribution to mathematics.

In the nineteenth century a number of Jesuit scholars from Europe spent part of their lives in the United States, and special mention should be made of Father **Johann G. Hagen** (1847–1930), a native of the Tyrol. He came to America in 1880, and after serving for a time as professor of science at the College of the Sacred Heart in Wisconsin he went to Georgetown College, remaining there as director of the observatory until called to be papal astronomer at the Vatican. Georgetown is the oldest Catholic college of scientific importance in the United States and since 1842 has ranked high as an astronomical center. Father Hagen, however, was more than an observer, for he contributed to mathematical periodicals and wrote a *Synopsis der höheren Mathematik*.[19] This publication was well received by such reviewers as Mansion, Neuberg, Moritz Cantor, and Stäckel, and ranks as a valued work of reference. Father Hagen also prepared an *Index operum Leonardi Euleri* (1896) which was of great assistance in rendering possible the new edition of Euler's works.

When Sylvester began his work at Johns Hopkins he numbered among his students **George Bruce Halsted** (1853–1922). After graduating at Princeton (1875) and taking his A.M. three years later, Halsted went to Johns Hopkins and received his Ph.D. in 1879. From 1884 to 1903 he was professor of mathematics at the University of Texas, from 1903 to 1906 at Kenyon

[19] Vol. I, Berlin, 1891; vol. II, *ibid.*, 1894; vol. III, *ibid.*, 1900–1905.

(Ohio) College, and from 1906–1912 at the Colorado State Teachers College (formerly State Normal School). His work on non-Euclidean geometry, including translations of Bolyai, Saccheri, and Lobachevsky, did much to make the theory known. He also translated Henri Poincaré's *Foundations of Science* and wrote a textbook and a large number of expository articles on geometry.[20]

Among the young men who were trained in Germany in the period 1880–1900 and who brought back the inspiration fostered by courses in such universities as Göttingen, Berlin, Jena, and Leipzig, **Frank Nelson Cole**[21] deserves special mention, not alone because of his contributions to the group theory and the theory of functions, but because of his long service as secretary of the American Mathematical Society. After taking his doctor's degree at Harvard (1886) he went to Germany for further study, especially under Klein. From 1888 to 1895 he gave courses at the University of Michigan, and from 1895 until his death he was professor of mathematics at Columbia. While in Michigan he published a translation of Eugen Netto's Theory of Substitutions[22] (Ann Arbor, 1892), a work which did much to create an interest in the theory of groups. He also published several important memoirs including the following: "On rotations in space of four dimensions" (*Am. J. M.*, XII, 181), "The linear functions of a complex variable" (*Annals*, V, 121) and "A contribution to the theory of

[20] See L. E. Dickson in the *Am. Math. Mo.*, I, 337–340, and A. M. Humphreys in *Science* (U. S. A.), LVI, 160–161.

[21] 1861–1926. See the *Bull. Am. M. S.* (XXXIII, 773), with a list of his writings and a biographical sketch by T. S. Fiske.

[22] *Substitutionentheorie und ihre Anwendung auf die Algebra*, Leipzig, 1882.

he General Equation of the Sixth Degree" (*Am. J. M.*, VIII, 265). In addition to these papers he published an mportant review of Klein's *Ikosaeder* (*ibid.*, IX, 45–61). With reference to his work in the theory of groups it nay be noted that he and G. A. Miller (at one time his pupil) emphasized the theory in the field of abstract groups, whereas the Chicago school, through the papers of E. H. Moore, H. Maschke, and L. E. Dickson considered the theory from the standpoint of linear groups also.[23]

**Eliakim Hastings Moore** stands out prominently as one of the men who first comprehended the significance of Sylvester's work at Johns Hopkins. He was born at Marietta, Ohio, January 26, 1862 and died in Chicago, December 30, 1932. His college and university work was done at Yale (A.B., 1883; Ph.D., 1885) and Berlin (1885–86). After a year as instructor at Northwestern Academy, two years as tutor at Yale, two years as assistant professor, and a year as associate professor at Northwestern, he was called to a professorship at Chicago (1892), becoming the head of the department in 1896. From the latter position he retired in 1931. He received honorary doctorates from Göttingen (1899), Wisconsin (1904), Yale (1909), Clark (1909), Toronto (1921), and Northwestern (1927). Coming to Chicago in his prime, he was influential in building up a mathematical faculty of unusual strength. To this faculty were called in 1892 Oskar Bolza and Heinrich Maschke, both scholars of German training, the result being that

[23] See J. Pierpont's address, *Bull. Am. M. S.*, XI, 136–152; L. E. Dickson, "Report on the progress in the theory of linear groups." *Ibid*, VI, 13–19.

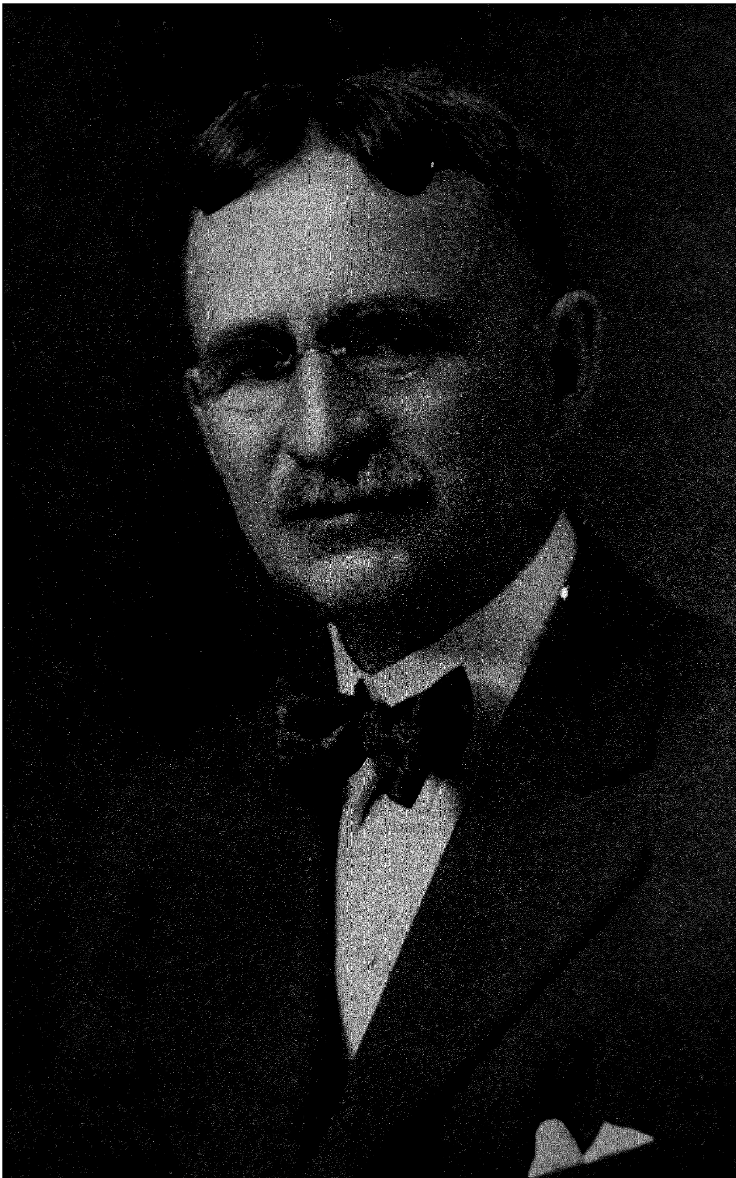
Chicago, although at that time in its infancy, promptly came to be looked upon as one of the leading mathematical centers. In his article on Professor Moore's life and works[24] Professor G. A. Bliss thus characterizes the trio of outstanding mathematicians in the early period of development of the Faculty:

> Moore was brilliant and aggressive in his scholarship, Bolza[25] rapid and thorough, and Maschke more brilliant but sagacious and without doubt one of the most delightful lecturers on geometry of all times. These three men supplemented one another perfectly, and they promptly obtained for the Department of Mathematics at the University of Chicago a place among the recognized leaders.

Moore's own interests were at first in geometry, groups, and the theory of functions, as witness his contributions to the *American Journal of Mathematics* (X, 17 and 243; XVIII, 264), the *Rendiconti del Circolo Matematico di Palermo* (IV, 186; IX, 86), the *Bulletin of the American Mathematical Society* (III (2), 11 and 73), the *Proceedings of the London Mathematical Society* (XXVIII, 357), and the *Mathematische Annalen* (XLIII, 271; LI, 417). After about 1900 he devoted himself chiefly to general analysis. Well did his colleague, L. E. Dickson, characterize the man (*Science*, LXXVII, (n.s.), 79): "Moore's work easily places him among the world's great mathematicians. In America, his various accomplishments made him a leader. But he was a leader who

[24] *The University Record*, Chicago, XIX, 130–134.

[25] Oskar Bolza (*b*. 1857) remained on the faculty for eighteen years, retiring in 1910 with the title of non-resident professor. He thereafter has made his home in Freiburg, Germany.



Eliakim Hastings Moore (1862–1932)

was universally loved, and this was because he was at the same time a prince of a man."

Among those whose names have been mentioned in connection with Moore's work at Chicago, **Heinrich Maschke**[26] was prominent for his mathematical contributions and for his influence with his students. His early education was acquired in the Gymnasium at Breslau and his university work was done at Heidelberg, Berlin, and Göttingen. His doctor's degree was secured in 1880 at the last-mentioned of these universities. After teaching in a Gymnasium for a time, he decided to prepare for a technical position, particularly in the line of electricity. This led him to further studies in Germany and to his coming to America in 1891. In 1892, however, he returned to an academic career as assistant professor of mathematics in the University of Chicago where he spent the last sixteen years of his life. Here he helped to build up the reputation of the mathematical faculty, bringing to his task the best traditions of Weierstrass, Klein, Königsberger, Kronecker, and Kummer. His lines of special interest were in finite groups of linear substitutions and in the theory of quadratic differential quantics, in the latter of which he developed a symbolic method of treatment to which he devoted much of his time in the years just preceding his death. His early reputation was made through memoirs appearing in the *Mathematische Annalen*—"Ueber die quaternäre, endliche, lineare Substitutionsgruppe der Borchardt'schen Moduln" (XXX, 496–515), "Aufstellung des vollen Formensystems einer quaternären

[26] Born at Breslau, Oct. 24, 1853; died in Chicago, March 1, 1907. See O. Bolza, *Bull. Am. M. S.*, XV, 85–95.

Gruppe von 51840 linearen Substitutionen" (XXXIII, 317–344), and "Ueber eine merkwürdige Configuration gerader Linien im Raume" (XXXVI, 190–215). His first important paper published in America was "The representation of finite groups, especially of the rotation groups of the regular bodies of three- and four-dimensional space, by Cayley's color diagrams" (*Am. J. M.*, XVIII, 156–188).

The study of the Theory of Functions received a new impetus in America through the arrival of **James Harkness** (1864–1923), a man who from his boyhood had shown unusual ability in the field of mathematics. At the age of eight he mastered the first book of Euclid's *Elements* without any help. He studied under Dr. E. J. Routh at Trinity College, Cambridge, graduating as eighth wrangler in 1885. In 1898 he came to Bryn Mawr College, and in the same year there was published the *Introduction to the Theory of Analytic Functions*, written in collaboration with Frank Morley of Johns Hopkins. The two authors had already published (London, 1893) *A Treatise on the Theory of Functions*, and both works had a great influence on the subject not alone in the British Commonwealth, but in the United States. Harkness and W. Wirtinger both contributed to the subject of elliptic functions in connection with R. Fricke's section (II, 2) in the *Encyklopädie der mathematischen Wissenschaften* (1913) and the more valuable French edition. He also wrote extensively on other branches of function theory.

Of the mathematicians whose years of activity crossed the dividing line of the preceding century and this, one

of the most gifted was **Maxime Bôcher.**[27] He was graduated at Harvard in 1888 and pursued his advanced studies in Germany. He was a prolific writer, his bibliography including 101 items, some of which are mentioned later. Quoting from Professor Birkhoff:

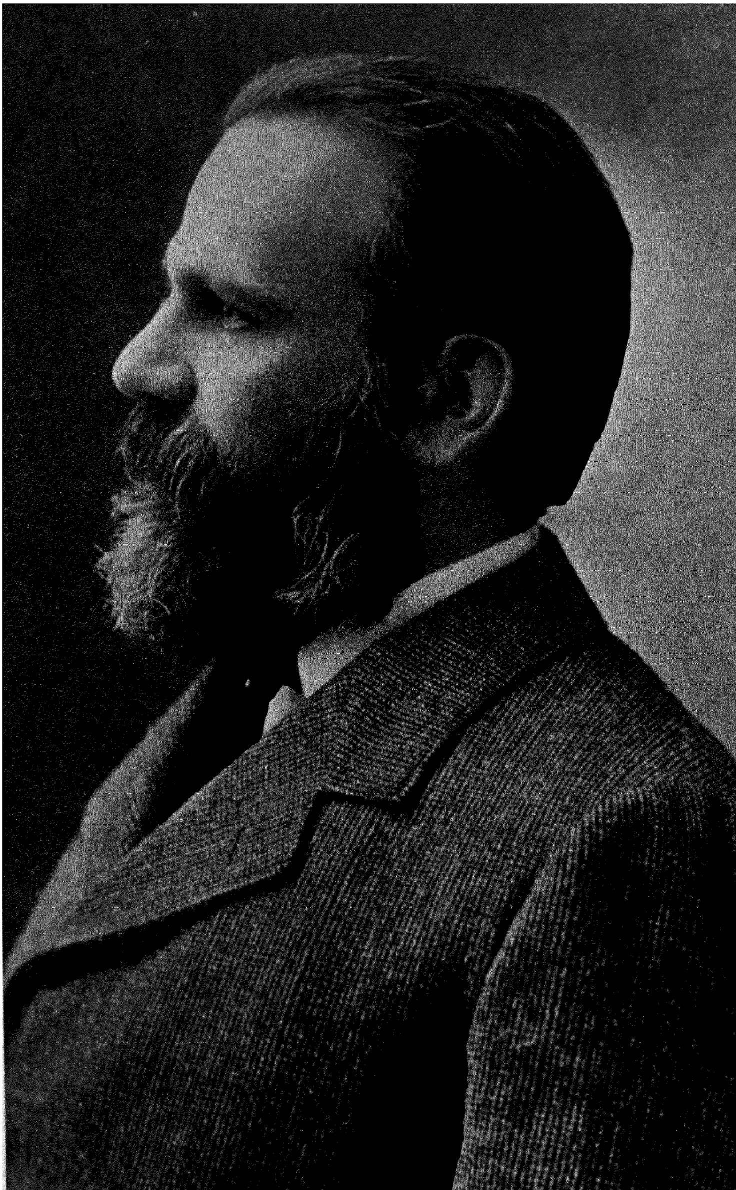
> His researches cluster about Laplace's equation $\Delta u = 0$ which is the very heart of modern analysis. Here one stands in natural contact with mathematical physics, the theory of linear differential equations both total and partial, the theory of functions of a complex variable, and thus directly or indirectly with a great part of mathematics . . . . His interest in the field of potential theory began in undergraduate days at Harvard University through courses given by Byerly and B. O. Peirce. An opportunity of penetrating further was given to him by his graduate work under Felix Klein at Göttingen (1888–1891).

It was at Göttingen that Bôcher wrote upon this subject, the result serving as his dissertation and at the same time as a prize essay, the title being "Ueber die Reihenentwickelungen der Potentialtheorie."

His interest in the history of mathematics is shown through two of his papers: (1) "A bit of mathematical history," *Bull. Am. M. S.*, II (2), 107–109; (2) Historical summary in Chapter IX, of W. E. Byerly's *Elementary Treatise on Fourier's Series and Spherical, Cylindrical and Ellipsoidal Harmonics* (Boston, 1893).

Of the mathematicians of the same period as that of Sylvester at Johns Hopkins, **Josiah Willard Gibbs** (1839–1903) is among those most widely known. This is rather strange since his original work lay in the field of physical chemistry rather than in that of mathe-

[27] *b.* Aug. 28, 1867; *d.* Sept. 12, 1918. See W. F. Osgood, "The life and services of Maxime Bôcher," *Bull. Am. M. S.*, XXV, 337–350; G. B. Birkhoff, *ibid.*, 197–215, with the bibliography of his contributions.

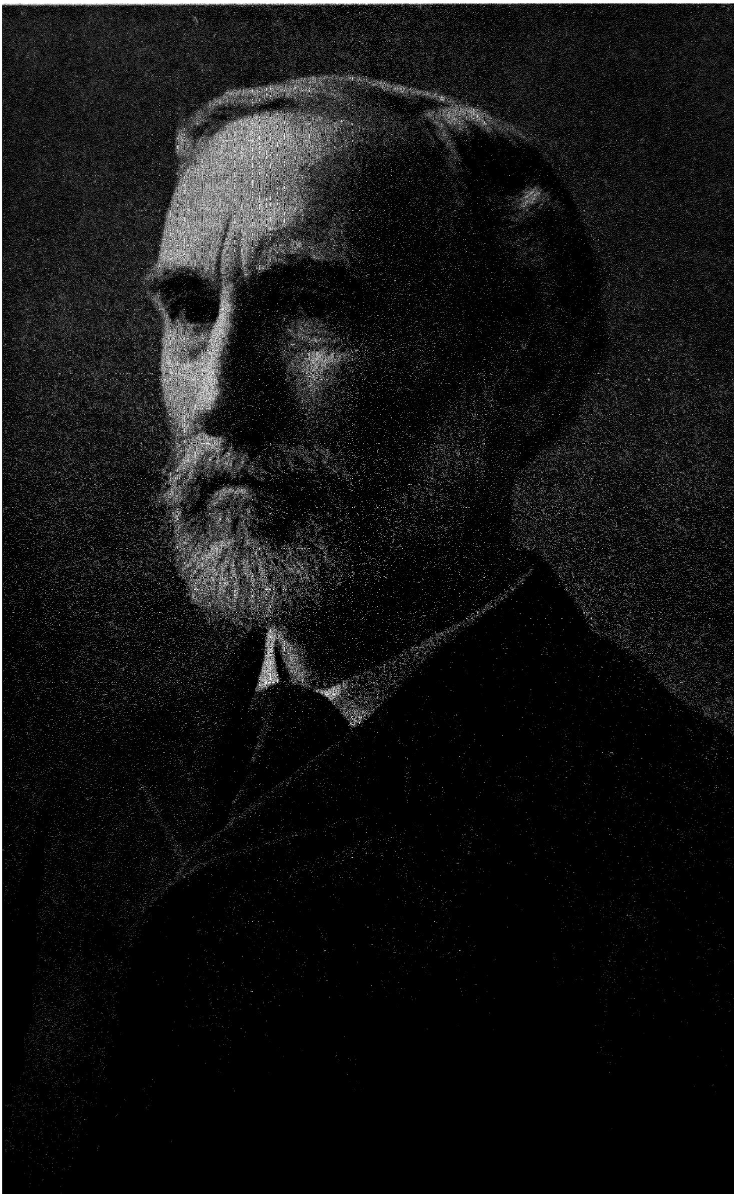


Maxime Bôcher (1867–1918)

matics as such. His creation of vector analysis in 1881—in which he discarded the quaternion symbolism of Hamilton, adopted some of the methods of Grassmann, and made use of the best of the work in complex numbers,—has given him a seemingly permanent place among the mathematicians of his century. This subject was first made known in print through some pamphlets published in 1881–1884 for the use of his students. In these he stated that the fundamental principles of the theory were those of quaternions, but that these were expressed in a simpler notation, thus making a departure from the Hamiltonian treatment—a departure for which he cites the precedent of Clifford's *Kinematics*. He also stated that his theory was in some respects more closely related to Grassmann's *Ausdehnungslehre* than to quaternions. At a considerably later date (1888) Edward Wyllys Hyde (1843–1930) published in the *Annals of Mathematics* (IV, 137–155) a paper on "The directional theory of screws" in which he also emphasized the superiority of the Ausdehnungslehre over quaternions, thus indirectly espousing the cause of vector analysis. The growth of the theory is considered later. See page 182.

## 6. AMERICAN DISSERTATIONS

The general trend of interest in the field of mathematics is shown in the subjects selected for doctor's theses in our universities in the closing years of the century, and also in the publications of papers by members of university faculties and by private scholars. We shall now consider the nature of the dissertations submitted in a few of the universities in which special



J. Willard Gibbs (1839–1903)

attention was given to mathematics and in which at least eight dissertations on mathematics were written during the period considered. These include only those in which the subjects and names are given under mathematics in the official lists of the several universities, even though others may appear elsewhere. They do not include those submitted in the departments of astronomy, physics, or other sciences, even though these may represent work in mathematics.

At Johns Hopkins there were thirty-two such dissertations in the period 1878–1900, as published in the university record, as follows:

Thomas Craig (1878), the representation of one surface upon another, and some points in the theory of the curvature of surfaces; George Bruce Halsted (1879), basis for a dual logic; Fabian Franklin (1880), bipunctual coordinates; Washington Irving Stringham (1880), regular figures in $n$-dimensional space; Oscar Howard Mitchell (1882), some theorems in numbers; William Pitt Durfee (1883), symmetric functions; George Stetson Ely (1883), Bernouilli's numbers; Ellery William Davis (1884), parametric representation of curves; Gustav Bissing (1885), some notes on Gauss's coordinates and Steiner's quartic surface; Henry Barber Nixon (1886), on Lamé's equation; David Barcroft (1887), forms of non-singular quintic curves; John Charles Fields (1887), symbolic finite solutions, and solutions by definite integrals of the equation $d^ny/dx^n = x^my$; Henry Taber (1888), on Clifford's $n$-fold algebra; William Curns Lawrence Gorton (1889), line congruences; Charles Hiram Chapman (1890), Riemann's P-function; Henry Parker Manning (1891), developments obtained by Cauchy's theorem with applications to the elliptic functions; George Frederic Metzler (1891), invariants and equations associated with the general linear differential equation; Daniel Alexander Murray (1893), associate equations of linear differential equations; Abraham Cohen (1894), a certain class of functions analogous to the theta functions; Lorrain Sherman

Hulburt (1894), a class of transcendental functions; Edward Payson Manning (1894), on the representation of a function by a trigonometric series; William Henry Maltbie (1895), on the curve $y^m - G(x) = 0$ and its associated Abelian integrals; René de Saussure (1895), sur la génération des courbes par roulement; Herbert Armistead Sayre (1896), the generation of surfaces; Thomas Hardy Taliaferro (1896), the focal surfaces of the congruence formed by the tangents to the lines of curvature of a given surface; Nathan Allen Pattillo (1897), certain partial differential equations connected with the theory of surfaces; Alexander Pell (1897), focal surfaces of the congruences of tangents to a given surface; John Eiesland (1898), a certain class of functions; James Graham Hardy (1898), on one-variable displacements in a space of four dimensions, and on curves of triple curvature; George Oscar James (1899), some differential equations connected with hypersurfaces; Luther Pfahler Eisenhart (1900), infinitesimal deformation of surfaces; Charles Ranald MacInnes (1900), superosculated sections of surfaces.

At Harvard there were nine such dissertations in the period 1873–1900, as follows:

William Elwood Byerly (1873), the heat of the sun; Frank Nelson Cole (1886), the general equation of the sixth degree; George Henry Johnson (1887), surfaces of minimum area; Joseph Lybrand Markley (1889), Bessel functions; James Waterman Glover (1895), functions defined by a certain partial differential equation; Milton Brockett Porter (1897), roots of hypergeometric and Bessel functions; Frederick Hollister Safford (1897), systems of revolution in the theory of Lamé's products; Donald Francis Campbell (1898), linear differential equations of the 3rd and 4th orders; Harry Yandell Benedict (1898), the variation of latitude.

At Yale University in the period 1875–1900 there were eighteen dissertations listed under mathematics as follows:

Mansfield Merriman (1876), on the elements of the method of

least squares; Joseph John Skinner (1876), on approximate computations; Andrew Wheeler Phillips (1877), on three-bar motion; Jefferson Engel Kershner (1885), on the determination of longitude between New Haven and Harvard College Observatory; Charles Newton Little (1885), on knots, with a census for order ten; Eliakim Hastings Moore (1885), on extensions of certain theorems of Clifford and of Cayley in the geometry of $n$ dimensions; Percy Franklin Smith (1891), on Plücker's complexes of lines; Charlotte Cynthia Barnum (1895), on functions having lines or surfaces of discontinuity; Jesse Breland Johnson (1895), on bicircular quartics; Elizabeth Street Dickerman (1896), on curves of the first and second degree in $x$, $y$, $z$, where $x$, $y$, $z$ are conics having two points in common; Shunkichi Kimura (1896), on general spherical functions; William Anthony Granville (1897), on the addition theorem in elliptic functions; George Tucker Sellew (1898), on the complex number; George Pratt Starkweather (1898), on the thermodynamic relations for water steam; Wendell Melville Strong (1898), on the necessity of continuity in Euclid's geometry; Jacob Westlund (1898), on some new equations of transformation; Leona May Peirce (1899), on chain differentiants of a ternary quantic; Herbert Edwin Hawkes (1900), on Peirce's Linear Associative Algebra.

In the period 1880–1900 there were eight dissertations at Columbia, as follows:

John Woodbridge Davis (1880), problems in applied mathematics; Winifred H. Edgerton (1886), multiple integrals; Samuel E. Stilwell (1886), the catenary; Thomas Scott Fiske (1888), the theory of concomitants of algebraic form; Edwin Mortimer Blake (1893), method of indeterminate coefficients and exponents applied to differential equations; George Herbert Ling (1896), on the solution of a certain differential equation which presents itself in Laplace's kinetic theory of tides; Edward Kasner (1899), the invariant theory of the inversion group; James Maclay (1899), on certain algebraic double minimal surfaces.

At Clark University in the period 1889–1900 there were twelve dissertations, as follows:

Jacob William Albert Young (1892), on the determination of groups whose order is a power of a prime; William Henry Metzler (1892), on the roots of matrices; Thomas Franklin Holgate (1893), on certain ruled surfaces of the fourth order; J. Wayland Dowling (1895), on the forms of plane quintic curves; Thomas F. Nichols (1895), on some special Jacobians; John E. Hill (1895), on quintic surfaces; Warren G. Bullard (1896), on the general classification of plane quartic curves; Frederick Carlos Ferry (1898), geometry of the cubic scroll of the first kind; Ernest W. Rettger (1898), on Lie's theory of continuous groups; John Shaw French (1899), on the theory of the pertingents to a plane curve; Frank Blair Williams (1900), on geometry on ruled quadric surfaces; Stephen Elmer Slocum (1900), on the continuity of groups generated by infinitesimal transformations; Morris Halcott Moreno (1900), on ruled loci in $n$-fold space.

At the University of Chicago in the period 1896–1900 there were eleven dissertations, as follows:

Leonard Eugene Dickson (1896), on substitutions on a power of a prime number of letters; discussion of the linear group; John Irwin Hutchinson (1896), on the reduction of hyperelliptic functions ($p=2$) to elliptic functions; Herbert Ellsworth Slaught (1898), on the cross-ratio group of 120 quadratic Cremona transformations of the plane; John Anthony Miller (1899), on elliptic modular functions of square rank; Gilbert Ames Bliss (1900), on geodesic lines on the anchor ring; George Lincoln Brown (1900), on a ternary linear substitution group of order 3360; William Gillespie (1900), on the reduction of hyperelliptic integrals ($p=3$) to elliptic integrals; Derrick Norman Lehmer (1900), on asymptotic evaluation of totient sums; John Hector McDonald (1900), on the system of a binary cubic and quadratic and the reduction of hyper-elliptic integrals to elliptic integrals; Ernest Brown Skinner (1900), on ternary monomial substitution groups; Forest Ray Moulton (1900),[1] periodic oscillating satellites.

[1] Degree in mathematical astronomy.

## 7. GENERAL TREND OF MATHEMATICS IN AMERICA, 1875–1900

In mathematics, as in natural science, philosophy, commerce, religion, government, architecture, and the habits and tastes of daily life, interests and fashions change from generation to generation. We have followed this change by an examination of the subjects of the dissertations of the period. We shall now continue the study by considering the other mathematical literature of the last two decades of the century.

Since the judgment of any one individual or of any committee would be open to personal bias, the selection of papers to be mentioned as most helpful in recording the progress of mathematics in America has been left generally to the judgment expressed in such works as the *Jahrbuch über die Fortschritte der Mathematik* during the generation preceding the twentieth century. Attention has also been given to the opinions expressed by more recent writers in articles appearing in such leading publications as the *Transactions of the American Mathematical Society*, the *American Journal of Mathematics*, and the *Encyklopädie der mathematischen Wissenschaften* (with the French translation and revision). The limited amount of space allowed in the present work has naturally made it necessary to omit certain references, but the lists which follow represent as fairly as possible the trend of mathematics in America during the period under consideration. In general all elementary textbooks, through the calculus, have been excluded, but more advanced treatises have been counted along with articles relating to mathematics beyond the ele-

ments, a certain amount of latitude being necessary in reaching a decision.

The first volume of the *Jahrbuch* referred to the publications of 1868, and only two important American items were mentioned—one by Newcomb on the lunar theory and a brief note by Asaph Hall on a theorem proposed by Gauss. There were two others of less importance—Bledsoe's *Philosophy of Mathematics* and Robinson's textbook on the calculus, but neither had much lasting value. No American periodicals were mentioned for the years 1868–1872; in 1873 the *Transactions* of the Connecticut Academy of Arts and Sciences was barely referred to, but was not listed at all in 1874; and it was not until the latter year that any attention was given to the contents of any scientific journal published in this country. In that year a summary of volume I of *The Analyst* appeared. In 1875 volume VII of the *Smithsonian Contributions to Knowledge* was mentioned for its publication of Simon Newcomb's memoir "On the general integrals of planetary motion;" but what is more significant was a summary of the mathematical articles which had appeared in *The Analyst* in volumes II–V. The question for us to consider, however, is the nature of the articles which were published in this country and which set what may be called the mathematical fashion from time to time. In the three American periodicals reviewed in this number (1875) of the Jahrbuch, twenty-six contributors published thirty-two papers, of which nine were on astronomy and geodesy, five were on number theory, and five were on analytic geometry, the rest being distributed among other topics. Only one was on series

and one on the theory of functions. Evidently astronomy was still the dominating influence in the mathematical field. This is shown not only in the articles but in the names of such contributors as Simon Newcomb, Ormond Stone, G. W. Hill and two or three others who were producing papers of importance in this field. In other lines there were Artemas Martin, a man possessed of considerable ability, who was writing on special problems in the number theory; W. W. Johnson, who was opening the door to a wider range of mathematics, and a few others who by their articles showed an interest in the modern branches of the science. Little of real importance was, however, manifest in the mathematical papers which were published prior to 1875.

In 1876 the list of American journals included for the first time the *Proceedings of the American Philosophical Society*, and in 1877 the *Journal of the Franklin Institute* was added. The mathematical contributions of American writers to their own and English publications continued, however, to refer in general to details of algebra and geometry rather than to the development of the newer branches. Thus, in 1876, twenty-four authors contributed forty articles, twenty of which referred to geometry, algebra, and the theory of numbers; astronomy, surveying, and mechanics claimed ten, and the rest were scattering, only one touching upon the function theory. Such topics as the extraction of roots, the prismoidal formula, rational right triangles, linkages, pasturage problems, and land surveying occupied more attention, or at any rate more space, than G. W. Hill's study of the problem of three bodies or his amplification of Delaunay's equation in the lunar

theory, or Newcomb's note on the longitude of the moon. One of the most hopeful features of the year was the beginning of more attention to the theory of probability, although this showed itself in only three items.

In 1877 the *Jahrbuch* reported twenty American contributors of forty-three articles, the general ratios of topics remaining about the same. Mechanics and astronomy account for about a fourth of the total; series and probability take about a fourth; and the calculus advances to about a fifth. This is the last report before the appearance of the *American Journal of Mathematics*, and it closes what may be called the period of preparation, a period of somewhat blind groping.

With this statement of the nature of the early work of this period we may now consider its representation by tabular means, dwelling later upon its significance. As already stated, the classification of articles in the *Jahrbuch* is not always satisfactory. In deciding the category in which to place a given article, for example, it is evident that different reviewers or editors may reach different conclusions. Nevertheless the results of a survey of the reviews for a series of years give a fair idea of the general progress of the subject. To avoid the fortuitous changes from year to year, due to the change of reviewers or of editorial assistants, three periods have been taken, beginning with 1875. The period 1868–1874 (volumes I–VI) is not included, the reason being that America was at that time hardly considered, only two volumes reporting any periodicals examined, and then only one in each year.

The first survey is for the three years, 1875–1877, these representing the period of the opening of Johns

Hopkins University and the beginning of the work of Sylvester. The second is the period 1878–1890, which represents approximately the first decade of real university work on a fairly large scale in America. The third is the last decade of the century when university work was approaching its present standard. In the table the term "Periodicals" means those published in America north of Mexico, and reviewed in the *Jahrbuch*. "Authors" means the average number of American authors publishing per year in American periodicals and whose articles were thought worthy of mention in the reviews, or of American authors publishing in the journals of other countries. "Articles" means the average number of articles so published, including solutions of problems above the elementary grade and treatises above the grade of elementary textbooks extending through the usual work in the calculus. Manifestly the number of periodicals is largely a matter of judgment of the *Jahrbuch* editors; and as this varies, so the number of authors and articles varies. Nevertheless the table tells at least a part of the story of the increase of interest and the value of the contributions in the field of mathematics.

| | 1875–1877 | 1878–1890 | 1891–1900 |
|---|---|---|---|
| Periodicals reported | 3.0 | 5.7 | 7.3 |
| Authors named | 23.0 | 31.2 | 59.6 |
| Articles and Treatises | 37.4 | 55.9 | 91.2 |

As to the topics of interest to mathematicians in the different periods it is difficult to make a satisfactory report because the classification in the *Jahrbuch* is not uniform. The following table gives, however, a fairly accurate statement of the results, with such adjustments as seem necessary under the circumstances. It

shows the average number of articles per year, as in the preceding table:

| | 1875–1877 | 1878–1890 | 1890–1900 |
|---|---|---|---|
| History, Biography, Philosophy, Teaching | 1.7 | 7.7 | 11.1 |
| Algebra | 3.3 | 9.8 | 20.4 |
| Theory of Numbers | 4.3 | 3.8 | 2.0 |
| Series | 3.0 | 1.8 | 2.8 |
| Calculus | 4.7 | 4.2 | 8.4 |
| Theory of Functions | 1.3 | 3.8 | 9.3 |
| Geometry | 6.7 | 13.2 | 20.4 |
| Mechanics and Mathematical Physics | 3.0 | 3.5 | 10.8 |
| Astronomy, Geodesy | 6.7 | 4.3 | 5.1 |
| Probability | 2.7 | 3.8 | 1.9 |

From this it appears that there was, in this period, a marked advance in the number of articles and treatises on pure mathematics in the fields of algebra, theory of functions, and geometry, and a fair advance in the differential and integral calculus. The *Encyklopädie der Mathematischen Wissenschaften* (Leipzig, 1898—) was also examined for the same purpose as the *Jahrbuch.* As a matter of some interest it may be mentioned that the smallest number of references to American authors, made in this valuable work, is in those parts dealing with the practical applications of mathematics. Those which refer to such abstract topics as determinants and curves have many more such references. Gibbs, however, is often mentioned.

In this connection it seems desirable to refer to the awakening, in the period 1875–1900, of an interest in the history of mathematics. This was probably due largely to a similar interest aroused by the works of such German scholars as Herman Hankel (1839–1873) and Moritz Cantor (1843–1920), knowledge of whose works began to reach America along with the general

influx of German mathematics already mentioned. Hankel was well known to American students in Germany through his *Vorlesungen über die komplexen Zahlen und ihre Funktionen* (Leipzig, 1867) and his posthumous work, *Die Elemente der projektivischen Geometrie* (Leipzig, 1875), and these naturally led to a knowledge of his historical works,—*Zur Geschichte der Mathematik in Alterthum und Mittelalter* (Leipzig, 1874) and *Die Entwickelung der Mathematik in den letzten Jahrhunderten* (Tübingen, 1869). Cantor's *Vorlesungen über Geschichte der Mathematik* began to appear in 1880 and exerted a great influence, especially in connection with his *Abhandlungen zur Geschichte der Mathematik* (Leipzig, beginning in 1877) and two earlier works. The influence of the French and Italian historians was not so marked, the number of American students in Europe being much greater in the Teutonic than in the Latin countries. The British influence of James Gow's *A Short History of Greek Mathematics* (Cambridge, 1884) showed itself rather markedly in our early histories, as did G. J. Allman's *Greek Geometry from Thales to Euclid* (Dublin, 1889), but far more influential were Sir Thomas Heath's publications which began with his *Apollonius of Perga* (1896) and his *Archimedes* (1897)[1]

Of the various American writers on the subject, Florian Cajori (1859–1930) was the first of any importance, and by his constant attention to the subject his influence was probably greater in developing it than

[1] His most important contributions, which include the second edition of the *Diophantus* (1910), *The Thirteen Books of Euclid's Elements* (1908), and *A History of Greek Mathematics* (1921), appeared after 1900 and therefore are not within the period discussed in this book.

that of any of his contemporaries. This was shown rather in a large number of brief notes than in his early histories. These notes showed painstaking research and form the kind of material from which histories are made. His most noteworthy contribution appeared after the period under consideration—*A History of Mathematical Notations* (Chicago, 1928, 1929), a fitting culmination of his life work. His earlier publications include *A History of Mathematics* (N. Y., 1894), *A History of Elementary Mathematics with Hints on Methods of Teaching* (N. Y., 1896), *A History of Physics* (N. Y., 1899), and *The Teaching and History of Mathematics in the United States* (Washington, 1890). Professor R. C. Archibald has listed 286 items in his bibliography of Cajori's articles, reviews, and books. (See *Isis*, XVII, 384–407.)

In 1896 there was published by M. Merriman and R. S. Woodward a work entitled *Mathematical Monographs* (New York). The first of these monographs was entitled "History of Modern Mathematics" and was written by D. E. Smith (81 pp., reprinted as a separate volume, New York, 1906), most of whose other works on the subject appeared after 1900. Of the various memoirs and addresses relating to the history of mathematics in this country or by American writers, appearing before that date mention should be made of the following: R. S. Woodward, presidential address before the American Mathematical Society, 1899; G. B. Halsted, various publications referred to elsewhere in this work; E. McClintock, "On the early history of non-Euclidean Geometry" (*Bull. N. Y. M. S.*, II, 144; see also p. 1); D. S. Hart, "History of American Mathematical Periodicals" (*The Analyst*, II, 131); W. W. Beman and D.

E. Smith, translation of K. Fink's *Geschichte der Mathematik* (Tübingen, 1890; transl., Chicago, 1900); J. G. Hagen, "On the history of the calculus" (*Bull. Am. M. S.*, VI, 381), and also his *Synopsis der höheren Mathematik* with its numerous historical and bibliographical notes.

In a historical sketch of this kind a brief statement should be made with respect to calculating machines. The early history of these devices relates to inventions in the seventeenth century in Great Britain (John Napier, 1617), France (Pascal, 1642), and Germany (Leibniz, 1671, 1694), and does not directly concern us. The eighteenth century saw various types that could be made cheaply and accurately enough to render their manufacture profitable. Their noteworthy progress began in the nineteenth century, especially in the Thomas machine (Colmar, Alsace, 1820) and the one by Burkhardt (Germany, 1878). The following are the more important American inventors of the period: Frederick P. Warren, who began his work on a calculating machine in 1858 and completed it in 1871 or 1872; he built a second and third model (1875) and died soon after the latter was completed, it having never been patented or put on the market; Frank Stephen Baldwin (1872), who filed a caveat for a four-process reversible calculating machine; he was the inventor of numerous others which ultimately developed into the Monroe; George B. Grant (1872), who built a differential engine and a calculating machine; E. D. Barbour, who patented the first mechanical multiplication table and a four-process calculating machine (1872, 1875); Ramon Verea (1878), who invented the first operable direct multiply-

ing machine, although this, like Barbour's devices, never reached the commercial stage; D. E. Felt (1885), who began the development of the first really practical key-drive adding machine, the comptometer; W. S. Burroughs (1885), who invented a listing machine that has had a large sale in all business houses; and the Patterson Brothers who, about 1883, developed the Cash Register type which is now used universally. Other types, including the popular Monroe (a combination of the numerous types devised by F. S. Baldwin and T. R. Monroe, 1913), the recent (1933) Lehmer machine for finding primes, the various tabulating and sorting systems devised by Hollerith about 1880, certain modern types of the slide rule, and devices for computations in physical laboratories, are products of the twentieth century.[2]

As already stated, the early achievements in mathematics in the New World were chiefly devoted to problems of astronomy. These involved careful observation, accurate computation, and the observance of certain established rules, together with elementary computations. While the names and certain achievements of various astronomers have been mentioned in the preceding chapters as being connected with the progress of mathematics, this ceases to be important after about 1850 except in the cases of a few men like Benjamin Peirce, G. W. Hill, and Simon Newcomb.

The departments of astronomy and physics, while depending more than ever upon mathematics, had so de-

[2] We are indebted to L. Leland Locke, who has given intensive study to the subject, for much of the above summary of the history of calculating machines in this country.

veloped by that date as to stand alone as distinct branches in the better type of colleges. For this reason the history of each of these subjects is now generally treated by itself and not as a topic in the history of mathematics. Such books as that of Dr. Solon I. Bailey, *The History and Work of Harvard Observatory, 1839–1927* will naturally be consulted as typical of the best records of work in astronomy in America in the period mentioned.

## 8. TREND OF IMPORTANT BRANCHES

As already remarked, the general trend of mathematics in any recent period may be seen in the subjects of the doctors' dissertations and in the publications of university professors and of private scholars. The dissertations having already been considered, it now remains to take account of the publications of those who, during the last quarter of the nineteenth century, directed the work in mathematics. For this purpose a list was compiled from the *Jahrbuch* and the *Encyklopädie* and from such mathematical periodicals as the *Transactions*, the *American Journal of Mathematics*, the *Annals*, the *Bulletin*, and the *American Mathematical Monthly*, together with similar publications in Europe.

This list, which excludes elementary textbooks and articles of similar grade, numbers upward of four hundred items, out of which in a brief survey of this kind it is possible to select only those deemed most important. Although the policy has been to exclude biographical sketches of living scholars, it is manifestly desirable to mention such of their papers as laid the foundations for the progress of mathematics in the present century.

In the new era, which considers mathematics in the large rather than as made up of isolated fields, the several domains may be expected to overlap, and hence certain papers which are classified under one head may be equally eligible for consideration under others.

(1) *Interest Shown in Algebra.* Taking algebra in the popular use of the term, as referring to a subject completed in the freshman year in college, no particular interest has been shown in its development during the past century. Admitting the desire to improve upon the methods of solving higher equations, the study of series, and the elementary treatment of the number systems, the trend has been to clarify each of these branches. If the domain were to be still further extended, we would have to consider the interest shown in the study of forms (quantics), groups, the calculus, determinants, quaternions and various other fields which will be given consideration later.

In the period in question the chief interest was shown in the solution of higher equations with such articles as the following:

**1883.** G. P. Young, "Principles of the solution of equations of the higher degrees, with applications," *Am. J. M.*, VI, 65–102; "Resolution of solvable equations of the fifth degree," *ibid.*, VI, 103–114.

**1884.** E. McClintock, "On the resolution of equations of the fifth degree," *Am. J. M.*, VI, 301–315.

**1886.** F. N. Cole, "A contribution to the theory of the general equation of the sixth degree," *ibid.*, VIII, 265–286.

More far-reaching was the tendency to search out the fundamentals of the subject, as in the following:

**1881.** B. Peirce, "Linear associative algebra," *Am. J. M.*,

IV, 97–229. The C. S. Peirce revision of the limited lithograph edition of a lecture delivered in 1870.

**1884.** J. J. Sylvester, "Lectures on the principles of universal algebra," *Am. J. M.*, VI, 270–286.

**1891.** H. B. Fine, "Kronecker and his arithmetical theory of the algebraic equation," *Bull. N. Y. M. S.*, I, 173–184.

O. Bolza, "On the theory of substitution groups and its application to algebraic equations," *Am. J. M.*, XIII, 59–144.

**1895.** M. Bôcher, "Gauss's third proof of the fundamental theorem of algebra," *Bull. Am. M. S.*, I, 205–209. See also *Am. J. M.*, XVII, 266–268.

**1896.** J. Pierpont, "Galois' theory of algebraic equations," *Annals*, I (2), 113–143; II (2), 22–56.

**1898.** J. Pierpont, "Early history of Galois' theory of equations," *Bull. Am. M. S.*, IV, 332–340.

E. B. Van Vleck, "On the polynomials of Stieltjes," *Bull. Am. M. S.*, IV, 426–438.

Other special cases, running into the theory of forms, are represented by the following:

**1879.** J. J. Sylvester, "On certain ternary cubic-form equations," *Am. J. M.*, II, 280–285 and 357–393; *ibid.*, III, 58–88 and 179–189.

**1881.** J. C. Glashan, "Forms of Rolle's Theorem," *Am. J. M.*, IV, 282–292.

**1892.** C. H. Kummell, "Symmetries of the cubic and methods of treating the irreducible case," *Annals*, VI, 179–197.

As to the school textbooks, they continued to emphasize the puzzle element, with little attention to the relation of algebra to arithmetic, geometry, or trigonometry. Their purpose seemed to be to lead to the manipulation of literal expressions of little direct interest.

(2) *Interest Shown in the Theory of Functions.* Concerning the advance in the theory of functions a reviewer in vol. 25 of the *Jahrbuch* discussed the preferences shown by the various nations in the field of mathe-

matics. He found that, in the years preceding the publication of Harkness and Morley's *Theory of Functions* (1893), the achievements of American and English writers were limited to the formal field (theory of forms, logical calculus, etc.), and so he welcomed the appearance of this work on the function theory. He expressed the belief that it showed an interest in an important branch which had been highly developed in France and Germany but neglected in the English-speaking countries.

Professor W. F. Osgood, speaking before a meeting of the Harvard Mathematical Club (Feb. 12, 1925) called attention, however, to the fact that James Mills Peirce had lectured on the subject in the seventies, and that his presentation of the theory was solely from the Cauchy standpoint, being founded on Briot and Bouquet's *Fonctions Elliptiques*. He called further attention to the fact that F. N. Cole championed the geometric treatment which Klein had given in his well-known Leipzig lectures of 1881–1882. Cole lectured at Harvard upon the subject for a short time before going to Michigan and may be considered an apostle of the new order, inaugurating a new era in graduate instruction in the latter university.

In a list of nearly fifty articles on the subject, published in the period under consideration, one third appeared in the seventeen years preceding the appearance of the Harkness and Morley work and two thirds in the eight years which followed. The relation of the subject to differential equations, quantics (forms), groups, geometry, and the calculus is such as to render exact classification difficult, but the list here given includes

enough of the more important papers to show the general trend of interest. In the line of elliptic functions the following may be mentioned:

**1877.** L. W. Meech, "Short method of elliptic functions," *Analyst*, IV, 129–136, 161–168; V, 9–16.

**1878.** C. H. Kummell, two papers supplementing the preceding, *ibid.*, V, 17–19, 97–104.

**1882.** Th. Craig, "Some elliptic function formulae," *Am. J. M.*, V, 62–75.

**1885.** W. E. Story, "The addition theorem for elliptic functions," *ibid.*, VII, 364–375.

**1889.** F. Franklin, "Note on the double periodicity of the elliptic functions," *ibid.*, XI, 283–292.

**1890.** E. H. Moore, "Note concerning a fundamental theorem of elliptic functions," *Rendiconti del Circolo . . . Palermo*, IV, 186–194.

**1891.** T. S. Fiske, "Weierstrass' elliptic integral," *Annals*, VI, 7–11.

J. H. Boyd, "An application of elliptic functions to a problem in geometry," *ibid.*, VI, 93–97, 163–165; see also VII, 1–10, 145–186.

**1893.** W. I. Stringham, "On the Jacobian elliptic functions," *ibid.*, VIII, 105–116. See also his paper in the *Chicago Congress Papers*, 1893, published in 1896.

The theta functions were considered by Th. Craig in the years 1882–1885 in the *American Journal*, V, 350–357; VI, 14–59, 337–358. On the general theory of functions, W. H. Echols published a number of papers about ten years later; see *Annals*, VII, 93–101, 109–142; VIII, 22–24, 45–51, *etc.*; *American Journal*, XV, 316–320; XVI, 207–220. W. F. Osgood also considered the general theory in the *Bulletin*, II (2), 296–302, and V, 59–87, with special cases in IV, 417–424 and elsewhere. F. N. Cole discussed "The linear functions of a complex variable" in the *Annals*, V, 121–176, and in 1896

T. S. Fiske wrote on the general theory in Merriman and Woodward's *Higher Mathematics*. Of the papers discussing the more special fields the following brief list will serve to show the general trend:

**1879.** J. J. Sylvester, "Excursus on the divisors of cyclotomic functions," *Am. J. M.*, II, 357–381. Part of a memoir already mentioned under Algebra.

**1889.** H. B. Fine, "On the functions defined by differential equations," *ibid.*, XI, 317–328.

**1891.** M. Bôcher, "On some applications of Bessel's Functions with pure imaginary index," *Annals*, VI, 137–160; see also 85–90.

**1893.** J. Harkness and F. Morley, *Treatise on the theory of functions*, New York, ix + 503 pp.

**1895.** O. Bolza, "On the first and second logarithmic derivatives of hyperelliptic $\sigma$-functions," *Am. J. M.*, XVII, 11–36.

J. Pierpont, "On the Ruffini-Abelian theorems," *Bull. Am. M. S.*, II (2), 200–221. His most important works on the theory of functions appeared after 1900 (Boston, 1905, 1912, 1914).

**1897.** M. Bôcher, "On certain methods of Sturm and their application to the roots of Bessel's functions," *Bull. Am. M. S.*, III, 205–213.

O. Bolza, "On Weierstrass' system of hyper-elliptic integrals of the first and second kinds," *Chicago Congress Papers*, 1–12, read in 1893, published in 1896.

J. McMahon, "Hyperbolic Functions," in Merriman and Woodward's *Higher Mathematics*, New York, 107–168.

E. B. Van Vleck, "On the roots of Bessel- and P-functions," *Am. J. M.*, XIX, 75–85.

**1898.** M. B. Porter, "On the roots of hypergeometric and Bessel's functions," *Am. J. M.*, XX, 193–214. See also *Bull. Am. M. S.*, IV, 274–275.

E. D. Roe, "On symmetric functions," *Am. Math. Mo.*, V, 1–6, 25–30, 53–58, 161–164.

**1899.** O. Bolza, "The partial differential equations for the hyperelliptic $\theta$- and $\sigma$-functions," *Am. J. M.*, XXI, 107–125;

"Proof of Brioschi's recursion formula for the expansion of the even $\sigma$-functions of two variables," *ibid.*, 175–190.

**1900.** W. F. Osgood, "On the existence of the Green function for the most general simply connected plane region," *Trans. Am. M. S.*, I, 310–314.

This last article, written before 1900, has attracted special attention, as in Professor H. Bateman's *Partial Differential Equations of Mathematical Physics* (1932). Speaking of the problem of conformal representation for the case of boundaries made up of segments of analytic curves, the latter remarks upon the pioneer work of H. A. Schwarz and the removal of certain restrictions by Osgood in this article. He also comments on the fact that this work has been followed up by many other investigators, including E. Study, C. Caratheodory, P. Koebe, and L. Bieberbach.

(3) *Interest Shown in Quantics or Forms.* The study of quantics or forms in America may be said to have been practically non-existent before Sylvester came to Johns Hopkins. In the last two decades of the nineteenth century there appeared a large number of papers upon the subject, generally in the *American Journal*, the *Annals*, or the *Bulletin*. The following abridged list will show the phases of the subject which occupied the most attention:

**1878.** J. J. Sylvester, "On an application of the new atomic theory to the graphical representation of the invariants and covariants of binary quantics," *Am. J. M.*, I, 64–90, a paper which, fifty years later, attracted renewed attention on the part of modern physicists. See also the "Note on M. Hermite's law of reciprocity," I, 90–118; "On Clebsch's theory of the 'Einfachstes System associirter Formen' and its generalization," I, 118–124; and "A synoptical table of the irreducible invariants and covariants of

a binary quantic," I, 370–378. The massing of this material in the first volume of the *Journal* had an immediate effect, especially upon the students and colleagues of Sylvester at Johns Hopkins.

**1879.** J. J. Sylvester and F. Franklin, "Tables of the generating functions and groundforms for the binary quantics of the first ten orders," *Am. J. M.*, II, 223–251; continued for "simultaneous binary quantics of the first four orders, taken two and two together," II, 293–306. See also 1881 and the *Johns Hopkins Circulars* for 1882, p. 178.

**1880.** F. Franklin, "On the calculation of the generating functions and tables of groundforms for binary quantics," *Am. J. M.*, III, 128–153.

**1881.** J. J. Sylvester, "Tables of the generating functions and groundforms of the binary duodecimic, with some general remarks, and tables of the irreducible syzygies of certain quantics," *ibid.*, IV, 41–61.

**1882.** F. Franklin, "On cubic curves," *ibid.*, V, 212–217.

J. J. Sylvester, "Tables of generating functions reduced and representative for certain ternary systems of binary forms," *ibid.*, V, 241–250. On partitions see pp. 251–330, with corrections by M. Jenkins, VI, 334–336, and VII, 74–81.

J. J. Sylvester, "On subinvariants," *ibid.*, V, 79–136.

**1885.** J. Hammond, "Syzygy tables for the binary quintic," *ibid.*, VIII, 19–25; "On the syzygies of the binary sextic and their relations," VII, 327–344.

**1886.** J. J. Sylvester, "Lectures on the theory of reciprocants," *ibid.*, VIII, 196–260.

**1888.** W. E. Heal, "On certain singularities of the Hessian of the cubic and the quartic," *Annals*, IV, 37–46.

**1891.** R. A. Harris, "On the invariant criteria for the reality of the roots of the quintic," *ibid.*, V, 217–228. Historical introduction by J. McMahon.

**1892.** H. S. White, "On generating systems of ternary and quaternary linear transformations," *Am. J. M.*, XIV, 274–282; "A symbolic demonstration of Hilbert's method for deriving invariants and covariants of given ternary forms," XIV, 283–290.

E. McClintock, "On lists of covariants," *Bull. N. Y. M. S.*, I,

85–91. "On the computation of covariants by transvection," *ibid.*, XIV, 222–229.

W. F. Osgood, "The system of two simultaneous ternary forms," *ibid.*, XIV, 262–273; "The symbolic notation of Aronhold and Clebsch," XIV, 251–261.

**1894.** R. A. Roberts, "On a certain class of canonical forms," *Bull. Am. M. S.*, I, 105–111.

H. B. Newson, "On a remarkable covariant of a system of quantics," *Bull. Am. M. S.*, II, 272–275.

H. S. White, "Semi-combinants as concomitants of affiliants," *Am. J. M.*, XVII, 235–265.

**1896.** C. H. Kummell, "To express the roots of the solvable quantics as symmetrical functions of homologues," *ibid.*, XVIII, 74–94.

H. Taber, "Notes on the theory of bilinear forms," *Bull. Am. M. S.*, III, 156–164.

H. S. White, "The cubic resolvent of a binary quantic derived by invariant definition and process," *ibid.*, III, 250–253.

**1897.** H. Taber, "On the group of linear transformations whose invariant is a bilinear form," *Math. Rev.*, I, 154–168. See also *ibid.*, I, 120–126, and *Proc. Am. Acad.*, XXXII, 77–83.

A. S. Hathaway, "On the transformations between two symmetric or alternate bilinear forms," *Math. Rev.*, I, 120–126.

**1898.** H. S. White, "Report on the theory of projective invariants: the chief contributions of a decade," *Bull. Am. M. S.*, V, 161–175.

(4) *Interest in Transformations.* Although finding a place under other branches of analysis, the last decade of the period here considered saw this subject attracting such a degree of attention as to render it desirable to list the following papers under one caption:

**1885.** C. H. Kummell, "The quadric transformation of elliptic integrals," *Bull. Washington Philos. Soc.*, VII, 102–121.

**1892.** H. S. White, "On generating systems of ternary and quaternary linear transformations," *Am. J. M.*, XIV, 274–282.

R. A. Harris, "Note on the use of supplementary curves in

isogonal transformations," *ibid.*, XIV, 291–300. See also *Annals*, VI, 77–80.

**1896.** E. O. Lovett, "Lie's geometry of contact transformations," *Bull. Am. M. S.*, III, 321–350; "Note on the general projective transformation," *Annals*, X, 5–16. See also XI, 33–47.

**1897.** E. O. Lovett, "Note on the infinitesimal projective transformation," *Bull. Am. M. S.*, IV, 515–519. See also 520–524, 402–404.

**1898.** M. W. Haskell, "On rational quadratic transformations," *Proc. Calif. Acad.*, I, 1–12.

L. E. Dickson, "The quadratic Cremona transformations," *Proc. Calif. Acad.*, I, 13–23.

E. O. Lovett, "Certain invariants of a quadrangle by projective transformation," *Annals*, XII, 79–86. The *Jahrbuch* for 1898 lists five of his papers on transformations.

A. Emch, "On circular transformations," *Annals*, XII, 141–160.

J. M. Page, "The general transformation of the group of Euclidean movements," *ibid.*, XII, 87–94.

H. B. Newson, "Normal forms of projective transformations," *Kansas Univ. Quarterly*, VIII, 43–66.

(5) *Interest Shown in the Calculus.* As with algebra and geometry, the calculus enters into all other branches of mathematics and the use of the term continually broadens. Hence it is feasible to mention only a relatively small number of articles under this caption, the elements having been developed before the period under discussion. The following references will serve to show, however, the general range of interest in certain features of the subject, especially in the last two decades of the century. The topic of differential equations is considered separately.

**1879.** E. McClintock, "An essay on the calculus of enlargement," *Am. J. M.*, II, 101–161. Other of his articles appear on pp. 307–314 and 348–353.

**1885.** L. B. Carll, *A treatise on the calculus of variations*, New York and London.

**1888.** M. Jenkins, "On Prof. Cayley's extension of Arbogast's method of derivations," *Am. J. M.*, X, 29–41.

**1889.** J. C. Fields, "The expression of any differential coefficient of a function of a function of any number of variables . . . ," *ibid.*, XI, 388–396.

**1891.** T. S. Fiske, "On certain space and surface integrals," *Annals*, VI, 61–63.

**1895.** E. McClintock, "Theorems in the calculus of enlargement," *Am. J. M.*, XVII, 69–80.

H. Hancock, "The Calculus of variations," *Annals*, IX, 179–190. See also *Annals*, XI, 20–32. The *Jahrbuch* for 1896 lists three of his articles on this subject.

**1898.** J. Pierpont, "Maxima and minima of functions of several variables," *Bull. Am. M. S.*, IV, 535–539.

W. F. Osgood, "The law of the mean and the limits of $0/0$, $\infty/\infty$" *Annals*, XII, 65–78.

W. H. Echols, "On circuit integration over a straight line," *ibid.*, XII, 175–181.

(6) *Interest Shown in Differential Equations.* The progress of linear differential equations before 1885 has been set forth by H. B. Nixon and J. C. Fields in the *American Journal of Mathematics* (VII, 353–363). In that period, as Professor Moulton has pointed out, G. W. Hill laid the foundations, in his memoir on the motion of the moon's perigee (1877), for the theory of homogeneous linear differential equations with periodic coefficients. In the same year W. W. Johnson wrote on the "Singular solution of differential equations of the first order" (*Analyst*, IV, 1–4). His continued interest in the subject led to his article in Merriman and Woodward's *Higher Mathematics* nearly twenty years later. The general increase of interest in the last two decades of the century may be inferred from the fact that about

two thirds of the articles on the subject appeared in the last five years of the period. The following are a few of the most important:

**1885.** Th. Craig, "On a certain class of linear differential equations," *Am. J. M.*, VII, 279–287. See also VIII, 85–103, 180–195.

**1887.** G. W. Hill, "On differential equations with periodic integrals," *Annals*, III, 145–155.

**1889.** Th. Craig, *A treatise on linear differential equations*, Vol. I (only), New York. In his review of the work in the *Bull. N. Y. M. S.*, I, 48–54, J. C. Fields calls it "the most advanced treatise on pure mathematics ever published by an American author."

**1890.** H. B. Fine, "Singular solutions of ordinary differential equations," *Am. J. M.*, XII, 295–322.

**1893.** Th. Craig, "Some of the developments in the theory of ordinary differential equations between 1878 and 1893," *Bull. N. Y. M. S.*, II, 118–134.

J. H. Boyd, "A study of certain special cases of the hypergeometric differential equation," *Annals*, VII, 145–186.

M. Bôcher, "On the differential equation $\Delta u+k^2u=0$." *Am. J. M.*, XV, 78–83.

**1895.** J. M. Page, "Transformation groups applied to ordinary differential equations," *Annals*, IX, 59–69.

**1896.** G. F. Metzler, "Equations and variables associated with linear differential equations," *ibid.*, XI, 1–9.

M. Bôcher, *Regular points of linear differential equations of the second order*, Cambridge, Mass., 22 pp.

**1897.** J. M. Page, *Ordinary differential equations . . . with an introduction to Lie's theory of the group of one parameter*, New York and London.

E. O. Lovett. The *Jahrbuch* for 1897 lists seven of his articles on differential equations. See also *Bull. Am. M. S.*, IV, 349–353, 457–487.

**1898.** M. Bôcher, "On singular points of linear differential equations with real coefficients," *Bull. Am. M. S.*, V, 275–281.

See also his articles in the *Annals*, XII, 45–53, and in the *Trans. Am. M. S.*, I, 40–52.

**1899.** E. B. Van Vleck, "On certain differential equations of the second order allied to Hermite's Equation," *Am. J. M.*, XXI, 126–167.

(7) *Interest Shown in the Theory of Numbers.* Interest in the theory of numbers is known to have existed in the days of the Pythagoreans, and probably many centuries earlier. The subject commanded the attention of such leaders as Fermat, Descartes, Gauss, Lagrange, Legendre, and other leaders of the seventeenth century and later. In the period which we are considering the articles of any length or of wide-reaching significance were few. Professor Dickson had not yet begun his *magnum opus* on the history of the subject, and to this we must refer for the American contributions as well as for those of the European scholars. As will be inferred from the following brief list, the number of important papers appearing between 1884 and 1899, inclusive, were few.

**1884.** W. W. Johnson, "Mr. Glaisher's factor tables and the distribution of primes," *Annals*, I, 15–23.

A. S. Hathaway, "Theory of numbers," *Am. J. M.*, VI, 316–331. Also see IX, 162–179.

**1887.** W. E. Heal, "Some properties of repetends," *Annals*, III, 97–103.

**1893.** E. W. Davis, "Geometrical illustrations of some theorems in number," *Am. J. M.*, XV, 84–90.

**1895.** F. Morley, "Note on the congruence $2^{4n} \equiv (-1)^n (2n)! (n!)^2$, where $2n+1$ is a prime," *Annals*, IX, 168–170.

E. H. Moore, "A two-fold generalization of Fermat's Theorem," *Bull. Am. M. S.*, II, 189–199.

**1896.** A. Martin, "About biquadrate numbers whose sum is a biquadrate," and "About cube numbers whose sum is a cube,"

*Math. Magazine*, II, 173–184, 185–190. Martin's contributions of this nature were numerous and generally appeared in his publications already mentioned. *E.g.*, *ibid.*, II, 200–208, 209–220.

L. E. Dickson, "Higher irreducible congruences," *Bull. Am. M. S.*, III, 381–389.

W. W. Beman's translation of Weber on transcendental numbers, *ibid.*, III, 174–195.

**1899.** L. E. Dickson, "A generalization of Fermat's Theorem," *Annals*, I (2), 31–36; *Comptes Rendus*, CXXVIII, 1083–1085.

(8) *Interest Shown in the Theory of Groups.* The growth of interest in the Theory of Groups is peculiar. The subject was little known in America until the closing decade of the nineteenth century. To be sure, Bolza had written an article, already referred to, published in the *American Journal of Mathematics* in 1891, but it was not until Cole, Moore, Maschke, and Dickson fresh from their studies in Germany, began to make it known, that American mathematicians awoke to its importance. The results are seen in the publication of more than sixty papers by American scholars in the last nine years of the century. The most prolific writer of the period was G. A. Miller, but the subject occupied the attention of Cole, Moore, Dickson, Page, Pierpont, and several others well into the present century. The following brief list will serve to show the general trend of the subject:

**1892.** F. N. Cole, *Theory of Substitutions* (being a translation of E. Netto's work of 1882), Ann Arbor.

F. N. Cole, "Simple groups from order 201 to order 500," *Am. J. M.*, XIV, 378–388. Carried to order 660 in XV, 303–315.

**1893.** F. N. Cole and J. W. Glover, "On groups whose orders are products of three prime factors," *ibid.*, XV, 191–220.

J. W. A. Young, "On the determination of groups whose order is a power of a prime," *ibid.*, XV, 124–178.

**1894.** J. M. Page, "Transformation groups in a space of four dimensions," *Annals*, IX, 11–22. See also VIII, 117–133.

**1895.** E. H. Moore, "Concerning Jordan's linear groups," *Bull. Am. M. S.*, II, 33–43.

**1896.** F. N. Cole, "On a certain simple group," *Chicago Cong. Math. Papers*, 40–43; read in 1893, published in 1896.

E. H. Moore, "A doubly-infinite system of simple groups," *Chicago Cong. Math. Papers*, 208–242. See also *Bull. N. Y. M. S.*, III, 73–78.

H. Maschke, "The representation of finite groups," *Am. J. M.*, XVIII, 156–194.

L. E. Dickson, "Systems of continuous and discontinuous simple groups," *Bull. Am. M. S.*, III, 265–273.

**1897.** L. E. Dickson, "On the structure of hypoabelian groups," *ibid.*, IV, 495–510.

G. A. Miller, "On transitive substitution groups," *Proc. Am. Philos. Soc.* (Philadelphia), XXXVI, 208–215. At about this time he contributed a large number of articles on the theory of groups. The *Jahrbuch* for 1896 lists ten of these articles; that of 1897, six; and that of 1898, fifteen. In 1898 he wrote a "Report on recent progress in the theory of groups of a finite order," *Bull. Am. M. S.*, V, 227–249.

**1898.** G. A. Miller, "On the primitive substitution groups of degree sixteen," *Am. J. M.*, XX, 229–241; "On perfect groups," *ibid.*, 277–282.

L. E. Dickson, "Systems of simple groups derived from the orthogonal group," *Proc. Calif. Acad.*, I, 29–40. The *Jahrbuch* for 1898 lists six of his articles on groups.

**1899.** G. A. Miller, "Memoir on the substitution groups whose degree does not exceed eight," *Am. J. M.*, XXI, 287–337.

Brief notes in the *Bull. Am. M. S.*, *Am. Math. Mo.*, and *Proc. Am. Philos. Soc.*

L. E. Dickson, "Determination of the structure of all linear homogeneous groups in a Galois field which are defined by a quadratic invariant," *Am. J. M.*, XXI, 193–256, with three articles in the *Bull. Am. M. S.*, and a "Report on the recent progress in the theory of linear groups," *ibid.*, VI, 13–27.

(9) *Interest Shown in Determinants.* Another field which was cultivated rather assiduously towards the close of the century was determinants. The first important step in this direction was taken by Benjamin Peirce in his *System of Analytical Mechanics* (Boston, 1855). In Chapter X, on "Determinants and Functional Determinants" he gives a brief account of determinants in general and then sets forth the main theorems of Jacobi's "De determinantibus functionalibus," rearranging the material and correcting certain statements. The subject also occupied the attention of J. E. Oliver in Runkle's *Mathematical Monthly* (III, 86–90), and was treated with more originality by John N. Stockwell in Gould's *Astronomical Journal*[1] (VI, 145–149), in connection with the resolution of symmetric equations with indeterminate coefficients.

Of the other contributors to determinants in various journals the following are among the most prominent: W. W. Johnson (*Analyst*, IV, 90), J. J. Sylvester (*Am. J. M.*), and G. W. Hill (*Acta Math.*, 1886; VIII, 1–36). Speaking of Hill's memoir Sir Thomas Muir[2] remarks, "This paper of Hill's is very different in character from those which precede it, and with it we seem to pass from matters of languid though legitimate curiosity to one of serious importance." L. Schlesinger[3] credits Hill with the introduction, in this paper, of the idea of a deter-

[1] This journal published a considerable number of mathematical articles, particularly on the perturbative function, the probability of error, higher plane curves, convergence of series, and analytic mechanics.

[2] *The Theory of Determinants in the historical order of development*, London, 1920, III, 467.

[3] *Handbuch der Theorie der linearen Differentialgleichungen*, Leipzig, 1895, I, xvi and 274.

ıinant of infinite order, and H. Poincaré with a simul-ıneous treatment of the subject in the *Bulletin* of the *ociété Mathématique de France*, (XIV, for 1886).

It being impossible in a brief survey of this kind to iscuss satisfactorily the nature of the contributions to ıe subject, it must suffice to give a list, compiled from Iuir, of the leading American contributors to the ͻneral theory and its principal branches in the period 880–1900, and not already mentioned, together with a rief supplement. In this period about 80% of the con-:ibutions were in English, French, or German. Of the rticles on determinants in general, as catalogued by Iuir, the number in English (37) predominated, venty-one being due to Muir himself, six to other ;ritish writers, and eleven to Americans, and those of ther countries whose articles appeared in American eriodicals. The names of these eleven given by Muir re E. W. Davis, H. T. Eddy, T. S. Fiske, J. W. Gibbs, .. S. Hathaway, E. O. Lovett, W. H. Metzler, E. Miller, '. Morley, N. M. Perry, and C. A. Van Velzer. The list ʼas not complete, however, for in the same period the ımerican writers of textbooks on the general theory in-lude P. H. Hanus (1886), W. G. Peck (1887), G. A. Iiller (1892), and L. G. Weld (1896 (in Merriman and Voodward's *Higher Mathematics*, 1896), to which hould be added a considerable number of authors of lgebras.

There were also Father J. G. Hagen, for the section ıevoted to the subject in his *Synopsis der höheren Iathematik*, and other writers of expository articles. n the field of axisymmetric determinants, to which both Iuir with nine articles and Sylvester with four gave

much attention, there were three articles contributed by other British writers and two by Americans (W. H. Metzler and H. S. White). On the topic of alternants there were 19 British and 12 American contributions, the latter being due to five authors,—O. H. Mitchell, Th. Craig, W. W. Johnson, W. H. Echols, and E. D. Roe. On compound determinants Muir mentions three papers by himself and five by three Americans—W. H. Metzler, H. S. White, and C. A. Van Velzer. The subject of recurrents seems to have been the first to attract special attention on the part of Canadian writers—J. B. Cherriman, N. F. Dupuis, and J. C. Glashan, the other four writers in English being residents of Great Britain.

To the other special off-shoots from this branch of mathematics American writers contributed liberally, especially to skew determinants, Pfaffians, orthogonants (W. H. Metzler), bigradients (W. E. Heal, M. W. Haskell, E. D. Roe, and E. B. Van Vleck), Hessians (F. Franklin), continuants (B. O. Peirce), multilineants (E. H. Roberts and E. W. Brown), $n$-dimensional determinants (E. R. Hedrick), and determinants with invariant factors (H. Taber). As to the product of $n$ determinants each of order $m$, and of $m$ determinants each of order $n$, discussed so fully by E. H. Moore in the *Annals of Mathematics* (I (2), 177–188), Muir remarks that the essential basis of Moore's paper is Metzler's important theorem of the preceding year.

From the above notes on the subject it is apparent that America showed great interest in determinants during the last quarter of the century—rather more than in other parts of algebra. The following brief list,

supplementary to Muir's as quoted above, will give some idea of the general trend of interest.

**1876.** D. Kemper, "Determinants," *Analyst*, III, 17–24.

**1882.** O. H. Mitchell, "Note on determinants of powers," *Johns Hopkins Circulars*, 1882, p. 242. See also *Am. J. M.*, IV, 341–344.

**1884.** C. A. Van Velzer, "Compound determinants," *Am. J. M.*, VI, 164–172.

**1890.** H. Taber, "On the theory of matrices," *Am. J. M.*, XII, 337–396; XIII, 159–172.

**1892.** W. H. Echols, "On certain determinant forms and their applications," *Annals*, VI, 105–126; VII, 11–59, 109–142.

W. H. Metzler, "On the roots of matrices," *Am. J. M.*, XIV 326–377. See other articles on determinants in *Am. J. M.*, XVI, 131–150; XX, 253–272, 273–276.

**1898.** E. B. Van Vleck, "On the determination of a series of Sturm's functions by the calculation of a single determinant," *Annals*, I (2), 1–13.

(10) *Interest Shown in Quaternions and Vector Analysis.* The publication of the first of the Gibbs papers on vector analysis occurred at a time when quaternions were beginning to attract more attention from men of high standing than had been the case since Sir William Rowan Hamilton wrote his well-known *Lectures* on the subject (1853), or even since the theory had first been made known in his papers read before the Royal Irish Academy in 1843.

In America, as elsewhere in the English-speaking world, the subject interested the physicists much more than it did the pure mathematicians, although for a time the latter made some attempt to adopt what seemed to many a cumbersome symbolism and to apply it to geometry. For example, in 1879, W. I. Stringham published an article on the quaternion formula for the

quantification of curves and surfaces (*Am. J. M.*, II, 205–210) and in the same periodical there appeared various other articles on the same subject.

The year 1881 saw the beginning of the conflict between the forces of quaternions and those of vector analysis. W. E. Byerly's work on the calculus (Boston, 1881) emphasized the use of complex numbers, and a reviewer of this and other books on the subject remarked that this "shows how the subject of quaternions is coming to the front, and the passage from the subject of these chapters to quaternions is but a short one." In the same year Arthur Sherburne Hardy (1847–1930), known rather as a diplomat and a novelist than as a mathematician, published his *Elements of Quaternions* (Boston, 1881) and a translation of Argand's memoir on imaginary quantities in which was included an historical account by J. Hoüel. In 1878 the *American Journal of Mathematics* (I, 379–383) published an article on "The Tangent to the Parabola," by M. L. Holman and E. A. Engler, in which the quaternion method was employed.

Thus the year 1881 seemed to find quaternions in a secure position. The theory of complex numbers may, however, lead in various directions, one being that of quaternions and another that of vector analysis and Ausdehnungslehre. For a number of years thereafter the quaternion notation held its own, with such advocates as James Byrnie Shaw who wrote on "The development of the A-process in quaternions" (*Am. J. M.*, 1897, p. 19), but who later did much to make known and appreciated the newer theory in his *Linear Associative Algebra* (1907) and *Vector Analysis* (1922).

In the decade following the early pamphlets of Gibbs

(1881) the innovation wrought by him aroused a rather animated controversy both in England and America. P. G. Tait, in the preface to his *Elementary Treatise of Quaternions* (3d. ed., Cambridge, Eng., 1890) took part with a spirit and in a manner easily understood from the following quotation:

> Prof. Willard Gibbs must be ranked as one of the retarders of the Quaternion progress, in virtue of his pamphlet on *Vector Analysis;* a sort of hermaphrodite monster, compounded of the notations of Hamilton and Grassmann.

But the "monster" won the battle. Among the defenders of vector analysis was Oliver Heaviside (1850–1925), particularly in his *Electro-dynamic Theory* (1893). As early as 1883, in his paper on "Some electrostatic and magnetic relations,"[4] he had taken a stand against the quaternion system saying that "there is great advantage in most practical work in ignoring quaternions altogether. . . . Professor Willard Gibbs, the author of a valuable work on Vector Analysis—has been denounced by Professor Tait in consequence as a retarder of quaternionic progress. Perhaps so, but there is no question as to the difficulty and the practical inconvenience of the quaternionic system."[5] The standing given to the theory by such writers as A. Föppl (1894), Max Abraham, M. A. Lorentz and other prominent scientists, led to a large number of textbooks on the subject, some of which are

[4] Reprinted from *The Electrician* (1883) in Heaviside's *Electrical Papers*, London and New York, 1892, I, 255–277.

[5] See also A. Ziwet's review of Wilson's *Vector Analysis*, *Bull. Am. M. S.*, VIII, 207–215, wherein he states that the Gibbs theory as set forth by Heaviside formed the basis, to a large extent, of Föppl's work of 1894.

listed in A. P. Wills's *Vector Analysis* (New York, 1931).[6]

A bibliography of quaternions up to 1903 was published by one of the leading champions of the subject, Alexander Macfarlane.[7] In this bibliography there are about thirty names of Americans by birth or residence. These include, with the years of publication, Wm. Beebe (1890), W. W. Beman (1881), M. Bôcher (1893), Ch. H. Chapman (1888–1895), E. B. Elliott (1887), H. B. Fine (1886–1890), J. W. Gibbs (1881–1902), M. L. Holman (1878), E. W. Hyde (1880–1902), W. W. Johnson (1875, 1880), Christine Ladd [Franklin] (1880), F. H. Loud (1893), A. Macfarlane (1883–1903, 28 items), W. H. Metzler (1893, 1894), G. A. Miller (1898), B. Peirce (1870–1881), C. S. Peirce (1875–1882), J. M. Peirce (1899), A. W. Phillips (1890), J. B. Shaw (1895–1909), G. P. Starkweather (1899, 1901), C. P. Steinmetz (1893–1901), W. I. Stringham (1878–1893), J. J. Sylvester (1883–1885), Henry Taber (1890, 1891), DeVolson Wood (1879–1881), R. S. Woodward (1895), and A. Ziwet (1878), showing the interest taken in the subject in the latter part of the nineteenth century.

Mention should also be made of A. Macfarlane's monograph, "The fundamental theorems of analysis generalized for space" (Boston, 1893, 31 p.), and A. McAulay's article, "Quaternions" (*Nature*, XLVII, p. 151), which dealt with the controversy between Heaviside and Gibbs.

[6] For a brief but pointed discussion of the contest see Nature, XLIII, 511–513, 535, 608; XLIV, 79–82; XLVII, 151. For lists of 16 later articles on the controversy see the *Jahrbuch*, XXV, 133 seq.

[7] Born in Scotland in 1851, in 1885 he became professor of physics in the University of Texas. He died in 1913. *Bibliography of Quaternions and Allied Systems of Mathematics*, Dublin, 1904.

The following is a list of a few more of the articles appearing in America and relating to the two theories:

**1877.** Christine Ladd [Franklin], "Quaternions," *Analyst*, IV, 172–174.

W. I. Stringham, "Investigations in quaternions," *Proc. Amer. Acad.*, XIII, 310–341.

**1879.** W. I. Stringham, "The quaternion formulae for quantification of curves, surfaces and solids, and for barycentres," *Am. J. M.*, II, 205–210. See also his "Determination of the finite quaternion groups," *ibid.*, IV, 345–347.

**1885.** A. Buchheim, "A memoir on biquaternions," *ibid.*, VII, 293–326.

**1892.** C. H. Chapman, "Application of quaternions to projective geometry," *ibid.*, XIV, 115–140.

**1894.** E. W. Hyde, "The screw as a unit in a Grassmann system of the sixth order," *Annals*, VIII, 38–44.

**1896.** E. W. Hyde, "Grassmann's space analysis," in Merriman and Woodward's *Higher Mathematics*, New York, 1896, pp. 374–424.

**1897.** J. B. Shaw, "The linear vector operator of quaternions," *Am. J. M.*, XIX, 267–282.

(11) *Interest Shown in Probability and Methods of Approximation.* Methods of approximation had attracted the attention of astronomers, physicists, and statisticians long before the period considered in this chapter. Adrain's proof of the theorem of Least Squares had suggested to American mathematicians the importance of the subject in the popular field of astronomy, and from that time to the present the question of approximations and probabilities has commanded the attention of some of our best astronomers, physicists, and statisticians.

The following are a few of the more important ar-

ticles and treatises on the subject, written in the period under consideration:

**1876.** F. H. Safford, "On the method of least squares," *Proc. Amer. Acad.*, IX, 193–201.

C. H. Kummell, "A new investigation of the law of errors of observation," *Analyst*, III, 133–140, 165–171. See also VI, 97–105 and 80–81; VII, 84–88.

**1877.** M. Merriman, "Notes on the history of least squares," *Analyst*, IV, 33–36, 140–143; an extensive bibliography, *Trans. Conn. Acad.*, IV, 151–232; *Elements of the method of least squares*, London.

**1879.** E. L. DeForest, "On unsymmetrical adjustments and their limits," *Analyst*, VI, 140–148, 161–170. See also 65–73 and VII, 1–9, 39–46, 73–82, 105–115; IX, 33–40, 65–79, 135–142, 161–168.

**1882.** R. S. Woodward, "On the actual and probable errors of interpolated values derived from numerical tables by means of first differences," *Analyst*, IX, 143–149, 169–175. Dr. Garver in his article on *The Analyst* already mentioned (page 115) remarks: "It seems to be the first serious paper written in this country on this important subject."

**1883.** E. L. De Forest, "On an unsymmetrical probability curve," *ibid.*, X, 1–7, 67–74. (On properties of polynomials see X, 97–105.)

**1891.** C. H. Kummell, "On an approximate method of computing probable error;" M. Merriman, "On the determination by least squares of the relation between two variables." These were both printed in an article by A. Macfarlane in the *Proc. Am. Assoc. Adv. Sci.*, XL, 53–103.

**1892.** W. W. Johnson, *The theory of errors and method of least squares*, New York, pp. x+174.

**1897.** G. H. Bryan, "On certain applications of the theory of probability," *Am. J. M.*, XIX, 283–288.

(12) *Interest Shown in Geometry.* Prior to 1875 little was done in geometry in America except as the subject was related to conic sections treated after the manner of

the Greeks or, after about 1850, analytically; or to Euclidean geometry, generally as rearranged by Legendre. Of the articles published in this country by Americans, only about a half dozen of much significance appeared between 1875 and 1880, and about twenty-five in the next decade; but in the period 1891–1900 at least seventy-five papers of some merit were published. Out of more than a hundred the following relatively small list of the more important will show the lines of interest in the subject:

**1876.** W. W. Johnson, "Recent results in the study of linkages," *Analyst*, III, 42–46, 70–74. The papers are valuable historically.

**1878.** G. B. Halsted, "Bibliography of hyperspace and non-Euclidean geometry," *Am. J. M.*, I, 261–276, 384–385; II, 65–70. The first important step in his history of a subject to the study of which he devoted much attention. See also a later report in *Science*, X (n.s.), 545–557.

**1879.** Christine Ladd [Franklin], "The Pascal hexagram," *Am. J. M.*, II, 1–12.

C. S. Peirce, "A quincuncial projection of the sphere," *ibid.*, II, 394–397.

**1880.** W. I. Stringham, "Regular figures in *n*-dimensional space," *ibid.*, III, 1–14.

Th. Craig, "Orthomorphic projection of an ellipsoid on a sphere," *ibid.*, III, 114–127.

W. W. Johnson, "The strophoids," *ibid.*, III, 320–325.

**1881.** Th. Craig, "The counter-pedal surface of the ellipsoid," *ibid.*, IV, 358–378. See also V, 76–78, and *Johns Hopkins Circulars*, 1882, pp. 209–210.

**1882.** W. E. Story, "Non-Euclidean trigonometry," *Am. J. M.*, IV, 332–335; V, 180–211. See also "On non-Euclidean properties of conics," *ibid.*, V, 358–382.

Th. Craig, *A treatise on projections*, Washington. Contains a history of the subject.

**1883.** E. W. Hyde, "Calculus of direction and position," *Am. J. M.*, VI, 1–13.

W. W. Johnson, "Circular coordinates," *Analyst*, X, 129–134.

C. H. Kummell, "Alignment curves on any surface, with special application to the ellipsoid," *Bull. Philos. Soc. Washington*, VI, 123–132.

**1886.** H. B. Fine, "On the singularities of curves of double curvature," *Am. J. M.*, VIII, 156–177.

**1888.** F. Morley, "On critic centres," *ibid.*, X, 141–148.

D. Barcroft, "Forms of non-singular quintic curves," *ibid.*, X, 131–140.

C. H. Chapman, "On some applications of the units of an $n$-fold space," *ibid.*, X, 225–242.

E. H. Moore, "A problem suggested in the theory of nets of curves and applied to the theory of the six points having multiply perspective relations," *ibid.*, X, 243–257; "Algebraic surfaces of which every plane section is unicursal," X, 17–28.

**1889.** F. Morley, "On the geometry of a nodal circular cubic," *ibid.*, XI, 307–316.

H. B. Newson, "On the eccentricity of plane sections of quadrics," *Annals*, V, 1–8.

**1890.** F. N. Cole, "On rotation in space of four dimensions," *Am. J. M.*, XII, 191–210.

F. Franklin, "On confocal bicircular quartics," *ibid.*, XII, 323–336; "On some applications of circular coordinates," XII, 161–190.

**1891.** F. Morley, "On the epicycloid," *ibid.*, XIII, 179–184.

M. W. Haskell, "Ueber die zu der Kurve $\lambda^3\mu+\mu^3\nu+\nu^3\lambda=0$ im projectiven Sinne gehörende mehrfache Ueberdeckung der Ebene," *ibid.*, XIII, 1–52.

W. C. L. Gorton, "On centres and lines of mean position," *Annals*, VI, 33–44.

G. B. Halsted, "Two-term prismoidal formula," *Scientiae Baccalaureus*, I, 169–178.

**1892.** C. P. Steinmetz, "On the curves which are self-reciprocal in a linear nulsystem," *Am. J. M.*, XIV, 161–186.

H. B. Newson, "Unicursal curves by method of inversion," *Kansas Univ. Quarterly*, I, 47–70.

W. W. Johnson, "Some theorems relating to groups of circles and spheres," *Am. J. M.*, XIV, 97–114.

C. P. Steinmetz, "Multivalent and univalent involutory correspondences in a plane determined by a net of curves of *n*th order," *ibid.*, XIV, 39–66.

**1893.** Charlotte A. Scott, "The nature and effect of singularities of plane algebraic curves," *ibid.*, XV, 221–243.

T. F. Holgate, "On certain ruled surfaces of the fourth order," *Am. J. M.*, XV, 344–386.

I. J. Schwatt, *A geometrical treatment of curves which are isogonal conjugate to a straight line with respect to a triangle*, Part I, Boston, 6+66 pp.

**1893.** H. B. Newson, "Linear geometry of the cubic and the quartic," Part I, *Kansas Univ. Quarterly*, II, 85–93. See also *Am. J. M.*, XIV, 87–94.

**1894.** F. Morley, "On adjustable cycloidal and trochoidal curves," *Am. J. M.*, XVI, 188–204.

**1895.** W. B. Smith, *Introductory modern geometry of point, ray and circle*, New York.

A. S. Chessin, "Geometrical multiplication of surfaces," *Annals*, IX, 70–72.

**1896.** W. F. Osgood, "A geometrical method for the treatment of uniform convergence and certain double limits," *Bull. Am. M. S.*, III, 59–86.

Charlotte A. Scott, "On Cayley's theory of the absolute," *ibid.*, III, 235–246.

**1897.** R. J. Aley, "Contributions to the geometry of the triangle," *Publ. Univ. of Penn.* (1897), 3–32.

H. B. Newson, "Types of projective transformations in the plane and in space," *Kansas Univ. Quarterly*, VI, 63–69; "On Hessians and Steinerians of higher orders," *Annals*, XI, 121–128; "Continuous groups of circular transformations," *Bull. Am. M. S.*, IV, 107–121.

V. Snyder, "Geometry of some differential expressions in hexaspherical coordinates," *Bull. Am. M. S.*, IV, 144–154.

H. S. White, "The construction of special regular reticulations on a closed surface," *ibid.*, IV, 376–382; "Inflexional lines, triplets and triangles associated with the plane cubic curves," *ibid.*, 258–260.

Charlotte A. Scott, "On the intersection of plane curves," *ibid.*, IV, 260–273. The *Jahrbuch* for 1898 lists four other articles.

**1898.** W. K. Palmer, "The hyperbolic spiral," *Kansas Univ. Quarterly*, VII, 155–172; "A graphical method of constructing the catenary," *ibid.*, 211–230.

T. F. Holgate, "A second locus connected with a system of coaxial circles," *Bull. Am. M. S.*, V, 135–143. Translation of Reye's *Lectures on the geometry of position*, New York.

G. F. Metzler, "Surfaces of rotation with constant measure of curvature and their representation on the hyperbolic (Cayley's) plane," *Am. J. M.*, XX, 76–86.

A. Pell, "On the focal surfaces of the congruences of tangents to a given surface," *ibid.*, XX, 101–134.

Th. Craig, "Displacements depending on one, two or three parameters in a space of four dimensions," *ibid.*, XX, 135–156.

A. Sayre, "The generation of surfaces," *Annals*, XII, 118–138.

J. I. Hutchinson, "The Hessian of the cubic surface," *Bull. Am. M. S.*, V, 282–292; "The asymptotic lines of the Kummer surface," *ibid.*, 465–467.

F. H. Safford, "Surfaces of revolution in the theory of Lamé's products," *ibid.*, V, 431–437.

**1899.** D. N. Lehmer, "Concerning the tractrix of a curve with planimetric applications," *Annals*, I (2), 14–20.

E. A. Engler, "The normal to the conic sections," *Trans. St. Louis Acad.*, VIII, 137–159.

(13) *Interest Shown by Foreign Contributors.* In the first half of the nineteenth century the general attitude of foreign scholars with respect to America was one of condescension. Beginning with 1880, however, this attitude changed and a new spirit of cooperation became evident. The following is an abridged list of the most important papers written by European mathematicians

and published in the United States during the next twenty years.

**1880.** Faà di Bruno, "Notes on modern algebra," *Am. J. M.*, III, 154–163. See also X, 169–172.

**1881.** A. Cayley, "On the analytical forms called trees," *ibid.*, IV, 266–268. See also articles by him in this *Journal* for a number of years.

**1883.** P. A. MacMahon, "Seminvariants and symmetric functions," *ibid.*, VI, 131–163. See also IX, 189–192, and later.

**1886.** H. Poincaré, "Sur les fonctions abéliennes," *ibid.*, VIII, 289–342.

**1887.** A. G. (later Sir George) Greenhill, "Wave motion in hydrodynamics," *ibid.*, IX, 62–112.

J. J. Sylvester (then at Oxford), "Lectures on the theory of reciprocants," *ibid.*, IX, 1–37, 113–161, 297–352; X, 1–16.

**1888.** R. Liouville, "Sur les lignes géodésiques des surfaces à courbure constante," *ibid.*, X, 283–292.

**1891.** P. Appell, "Sur les lois de forces centrales faisant décrire à leur point d'application une conique quelles que soient les conditions initiales," *ibid.*, XIII, 153–158.

F. Brioschi, "Sur une forme nouvelle de l'équation modulaire du huitième degré," *ibid.*, XIII, 381–386.

M. d'Ocagne, "Quelques propriétés des nombres $K_m^p$," *ibid.*, 145–152.

Joseph Perrott, "Remarque au sujet du théorème d'Euclide sur l'infinité du nombre des nombres premiers," *ibid.*, 235–308.

**1894.** E. Study, "On the connection between binary quartics and elliptic functions," *Bull. Am. M. S.*, I, 6–10; "On the addition theorem of Jacobi and Weierstrass, *Am. J. M.*, XVI, 156–163.

**1895.** E. Study, "On irrational covariants of certain binary forms," *Am. J. M.*, XVII, 185–215.

Ch. Hermite, "Sur la logarithme de la fonction gamma," *ibid.*, XVII, 111–116.

**1897.** G. Koenigs, "Sur un problème concernant deux courbes gauches," *ibid.*, XIX, 259–266.

S. Kantor, "Ueber Collineationsgruppen an Kummerschen Flächen," *ibid.*, XIX, 86–91.

R. de Saussure, "Calcul géomètrique réglé," *ibid.*, XIX, 329–370.

**1898.** Th. Muir, "An investigation of the problems of the automorphic linear transformation of a bipartite quadric," *ibid.*, XX, 215–228.

It will be observed that most of the papers in the above list were published in the *American Journal of Mathematics*, at that time the leading medium in this country for the dissemination of the results of advanced research. The interest of foreign scholars in this journal becomes even more apparent from the following list of writers, the figures in parentheses showing the numbers of the volumes in which the contributions appeared:

P. Appell (13), O. Bolza (11, 13), F. Brioschi (13), A. Cayley (1–15), W. K. Clifford (1), Faà de Bruno (3, 5, 10), A. R. Forsyth (12), A. G. Greenhill (9), C. Hermite (4, 17), E. Jahnke (21), Gomes-Texeira (7), S. Kantor (18, 19, 23, 24), G. Koenigs (19), L. Koenigsberger (11), R. Liouville (10), P. A. MacMahon (4, 9, 11–14), Th. Muir (20), M. d'Ocagne (9, 11, 13, 14), G. Peano (17), J. Perrott (13), J. Peterson (2), E. Picard (16, 20), H. Poincaré (7, 8, 12, 14), R. de Saussure (17, 18, 19), E. Study (16, 17), and G. Veneziani (7), to which list must, of course, be added the name of Sylvester, who continued to contribute after his return to England.

(14) *Interest Shown by Foreign Journals.* The value which foreign scholars began to place upon the work of American mathematicians in the last decade of the nineteenth century is further indicated by the rise in the number of articles of the latter published, for example, in the *Mathematische Annalen* in that period. Volumes 1–26 contain no such articles; volumes 27–34 (1886–1889) contain seven by Bolza and Maschke who were soon to come to Chicago to assist in placing that

university among those of the first rank; and volumes 41–50 (1893–1897) contain fifteen including the following:

Fabian Franklin (1893, then in Johns Hopkins) on Riemann's theory of Abelian functions; W. E. Story (1893) on quantics; O. Bolza on the group of an equation (1893), on the Riemann differential equations (1893), on transformations of the third order of elliptic functions (1898), and on hyperelliptic integrals of the first order (1898); H. Maschke on linear homogeneous substitutions (1898) and linear substitution groups (1898); E. H. Moore on triple systems (1894), on transcendentally transcendental functions (1897), on linear substitutions (1897), and on Abelian regular transitive triple systems (1897); M. Bôcher (1894) on projective reflections; Henry Taber (1895) on the automorphic linear transformation of an alternate bi-linear form.

The publication of American material in European journals was, of course, not limited to those of any single country like Germany or any single periodical like the *Annalen*. American students, especially those from Canada, naturally tended to go to England before about 1875 quite as much as to Germany. For linguistic reasons they tended to publish more freely in the former country than in the latter, even after that date. Thus we find in the *Quarterly Journal of Pure and Applied Mathematics* no American contributions before 1882, three articles on elliptic functions and alternants by W. W. Johnson in the period 1882–1886, two by Frank Morley on the triangle (1890–1891), and contributions by Harris Hancock on Kronecker modular systems (1895), J. C. Glashan on Sylvester's tables of Hamiltonian differences (1895), L. E. Dickson on number theory and groups (1895, 1898, 1899), Charlotte A. Scott on geometry (1896, 1898), E. O. Lovett on perturbations

and contact transformations (1899), and F. N. Cole (1893, 1895) and G. A. Miller (1895, 1896, 1898, 1900) on groups, with others in various fields.

Owing to the number of Americans enrolled in the London Mathematical Society it was natural that many of their papers should be published in its *Proceedings*. The following are some of the important ones which appeared in the last two decades considered in this work:

G. S. Ely on the solution of the modular equation for the septic transformation (1881); W. W. Johnson on systems of formulae for the *sn*, *cn*, and *dn* of $u+v+w$ (1881); H. Taber on matrices (1890), transformations between two quadrics (1892), on orthogonal substitutions (1894), and on Stieltjes' Theorem (1895); W. E. Story on an operator that produces all the covariants and invariants of any system of quantics (1891); E. H. Moore on groups (1896); E. O. Lovett on anharmonics (1897) and space of $n$ dimensions (1899); F. Morley on the Poncelet polygons of a limaçon and on a regular rectangular configuration of ten lines (1897); E. W. Brown on Delaunay's canonical system of equations (1895) and the lunar theory (1896); W. H. Metzler on symmetric functions (1896) and determinants (1898); L. E. Dickson on groups (1898, 1899, 1900); W. I. Stringham on non-Euclidean space (1900); and G. A. Miller on groups (1897, 1898, 1899, 1900).

The above lists show the nature of the contributions made by Americans to three prominent European journals during the period under discussion. In order to show more clearly the trend of mathematics and the range of periodicals the following selected list is added. This is arranged chronologically and includes five other journals.

**1882.** Th. Craig, "On the parallel surface to the ellipsoid,"

*Journ. reine und angewandte Math.*, XCIII, 251–270. See also XCIV, 162–170.

**1890.** F. Morley, "On the kinematics of a triangle of constant shape and varying size," *Quarterly Journ.*, XXIV, 359–369; 386 note.

**1891.** F. Morley, "Quaternions and the Ausdehnungslehre," *Nature*, XLIV, 79–82. See also XLIII, 511–513.

**1892.** W. E. Story, "On the covariants of a system of quantics," *Math. Annalen*, XLI, 469–490.

**1893.** F. Franklin, "Bemerkung über einen Punkt in Riemann's Theorie der abel'schen Funktionen," *ibid.*, XLI, 308.

F. N. Cole, "List of the substitution groups of nine letters," *Quarterly Journ.*, XXVI, 372–388. See also XXVII, 39–50.

J. C. Fields, "The numbers of sums of quadratic residues and of quadratic non-residues respectively taken $n$ at a time and congruent to any given integer to an odd prime modulus $P$," *Journ. reine und angewandte Math.*, CXIII, 247–261.

**1895.** E. H. Moore, "Concerning triple systems," *Math. Annalen*, XLIII, 271–285. See also *Rendiconti Circ. Mat. di Palermo*, IX, 86; *Proc. Lond. Math. Soc.*, XXVIII, 357–386, and various other papers.

H. Hancock, "On the reduction of Kronecker's modular systems," *Quarterly Journ.*, XXVII, 147–183.

G. A. Miller, "Intransitive substitution groups of ten letters," *ibid.*, XXVII, 99–118. See also XXX, 243–263; *Comptes Rendus*, CXXVI, 1406–1408, and various other papers.

L. E. Dickson, "Cyclic numbers," *Quarterly Journ.*, XXVII, 366–377.

**1896.** H. Taber, "On the automorphic linear transformation of an alternate bilinear form," *Math. Annalen*, XLVI, 561–583.

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## 9. RETROSPECT

In considering the development of mathematics in this country, before the twentieth century, a few definite tendencies are seen to characterize various epochs. From 1500 to 1600 the aims and the achievements were hardly commensurate with those of a mediocre elementary school of our time. From 1600 to 1700, they were not even equal to those of our high schools of low grade, but the purpose was more definite than in the preceding

century. The objective was now to give to those seeking it enough work in astronomy to predict an eclipse and to find the latitude of a ship at sea, and enough mensuration to undertake the ordinary survey of land.

From 1700 to 1800 the general nature of the work in the colleges was that found in the two great universities of England, but it was far from being of the same quality. The courses then began to include algebra, Euclid, trigonometry, calculus, conic sections (generally by the Greek method), astronomy, and "natural philosophy" (physics). The prime objective was still astronomy. Little advance was made upon the mathematics of England as taught in the preceding century, except that the sextant replaced the quadrant and astrolabe for purposes of navigation. In general the tendency was toward the application of mathematics to astronomy and natural philosophy. This union of these subjects with pure mathematics was a healthy one for all three branches, since none of them was sufficiently developed by itself to make a separate course practicable even if desirable.

From 1800 to 1875 America began to show a desire to make some advance in both pure and applied mathematics, independently of European leadership. The union of mathematics, astronomy, and natural philosophy was still strong, and the pursuit of mathematics for its own sake was still somewhat exceptional.

From 1875 to 1900, however, a change took place that may well be described as little less than revolutionary. Mathematics tended to become a subject *per se*; it became "pure" mathematics instead of a minor topic taught with astronomy and physics as its prime objec-

tive. American scholars returning from Europe brought with them a taste for abstract mathematics rather than its applications. There were naturally many exceptions, a subject like quaternions, for example, having necessarily to carry with it a considerable knowledge of mechanics; and differential equations having a wide range of applications. Neverthelesss the tendency was strongly toward pure analysis and geometry.

The question then arose as to whether this tendency was a healthy one, either for mathematics or the natural sciences. The answer lies with the generation following ours. All we can now say is that the pendulum was motionless in the sixteenth century; that it began to swing toward the applications in the seventeenth and eighteenth centuries; that it reached the limit of oscillation in that direction in the first half of the nineteenth century; that about 1875 it had definitely begun to swing to the side of pure mathematics; and that in the period ending with 1900 it seems to have reached the limit of the movement in that direction. Has the tendency now changed? Does mathematics reach out once more to a closer union with physics, celestial mechanics, the quantum theory, and the material ranges of the sciences in general? It is for the historian of the year 1950 to survey the first quarter of this century and then to answer this question and to venture, if he feels it safe, to prognosticate for the same length of time in the future.

In connection with the survey which can then be made, the historian will have a greater freedom than a contemporary has to pay the tribute due to men now living who, during the closing years of the nineteenth

century contributed so greatly to the remarkable development of mathematics in our present generation. The names of a number of these scholars have been mentioned under the topics of their special interests, but no critical evaluation of their work has been attempted and no biographical notes have been given for them. Most of these men will then have passed away and the historian, as he undertakes to analyze the mathematics of America as a whole, will reveal in proper perspective the rôles which men now living played in the half century beginning about 1880 and reaching to approximately the present time.

INDEX

Note: References are not generally given when names or subjects are mentioned only casually. They are reserved for such as relate to some mathematical contribution or to some important historical event.

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